Time series analysis

Plenary Lecture

Week 2.4, 1 Dec 2025

Christian Tiberius

Modelling, Uncertainty and Data for Engineers

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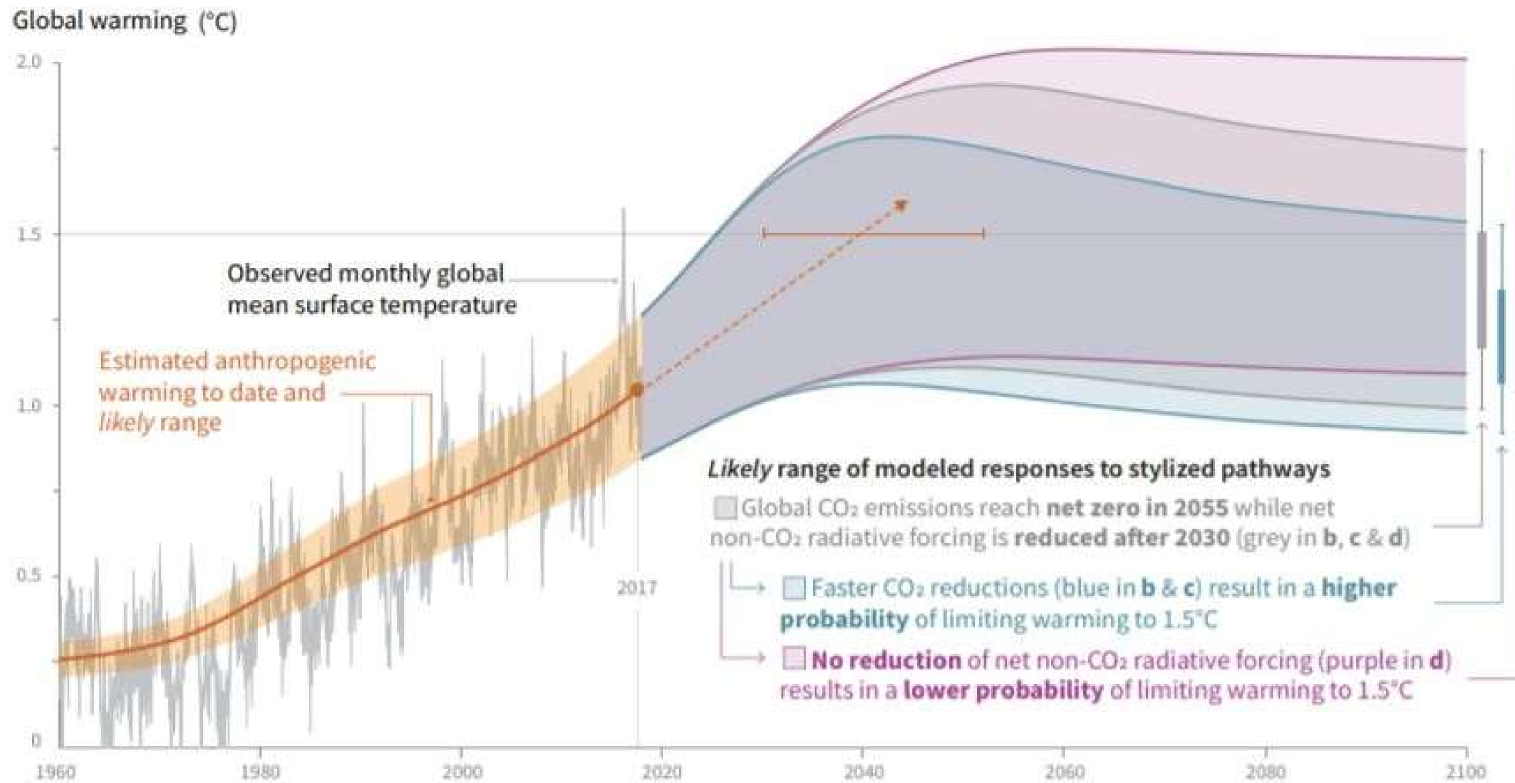
Time Series Analysis – an example



IPCC (Intergovernmental Panel on Climate Change), 2018. *Global Warming of 1.5° C. An IPCC Special Report on the impacts of global warming of 1.5° C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty*. Geneva. https://www.ipcc.ch/sr15/

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Time Series Forecasting

‘It’s hard to make predictions,
especially about the future’
Yogi Berra (1925-2015)

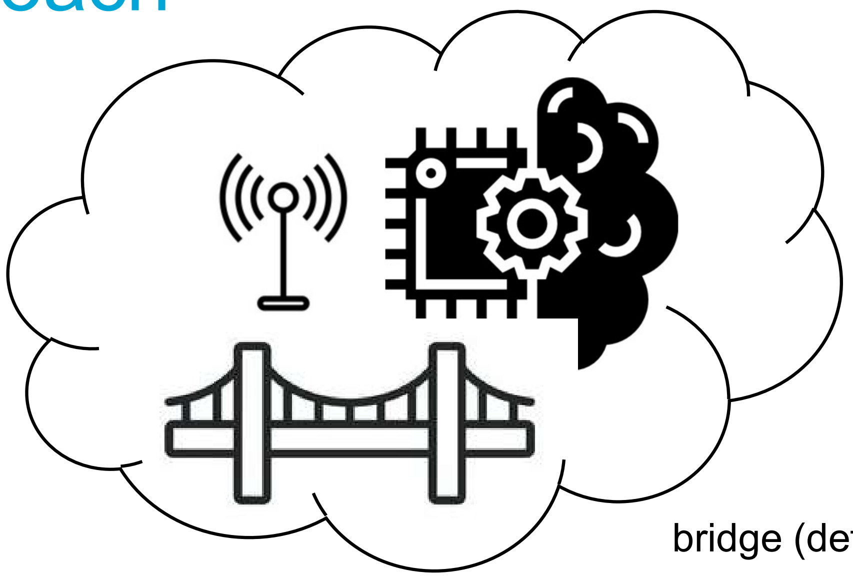
‘Kijk nooit verder dan je neus lang is... en je neus is maar drie dagen lang’
Jan Pelleboer (1924-1992)

https://www.youtube.com/watch?v=488oJaraNF4

Time series **analysis** comprises methods for analyzing time series data in order to extract meaningful statistics and other characteristics of the data.

Time series **forecasting** is the use of a model to predict future values based on previously observed values.

Modeling approach



bridge (deformation) monitoring system

mathematical model (of system)

functional model

$$\mathbb{E}(Y) = \mathrm{Ax} \quad \text{or} \quad Y = \mathrm{Ax} + \epsilon$$

random error

stochastic model

$$\mathbb{D}(Y) = \Sigma_Y = \Sigma_\epsilon$$

(and actually full statistical distribution of observable *Y*)

$$\epsilon \sim N(0, \Sigma_\epsilon)$$

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Time Series Analysis

time series: $Y(t) = [Y(t_1), Y(t_2), \ldots, Y(t_m)]^T$

continuous-time phenomenon observed/sampled at *m* instants

approach Time Series Analysis from Observation Theory perspective (week 1.7)

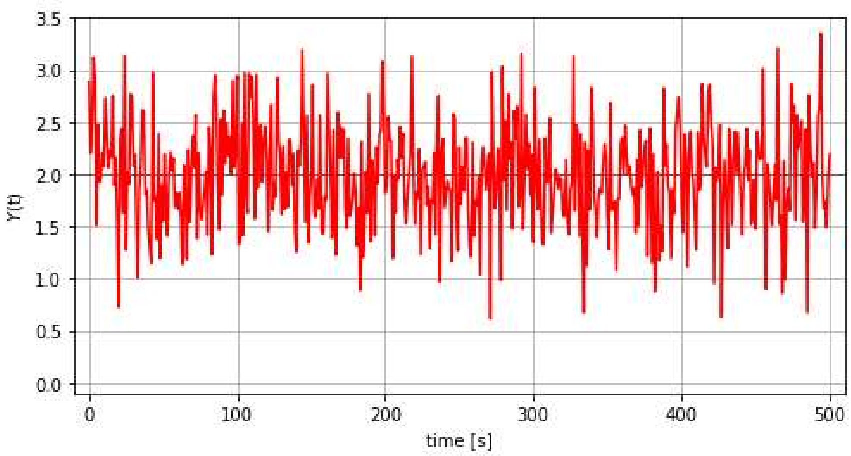
↓

make inferences so as to describe physical reality

$Y = \text{signal} + \text{noise}$

observable

noise: random, uncontrolled fluctuation of time series about its functional pattern



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mind, in week 2.3 on Signal Processing, we omitted noise,
there we worked, in principle, with *deterministic* signals/observables

Time Series Analysis

mathematical model (of system)

functional model

stochastic model

ideally, all functional effects included
(all mechanisms in the system modelled)

then, white Gaussian noise left

'just really random noise'

in practice: model is an approximation of reality, at best

then, stochastic model should
capture the left-overs …

you may see some patterns in the noise,
in particular time correlation!

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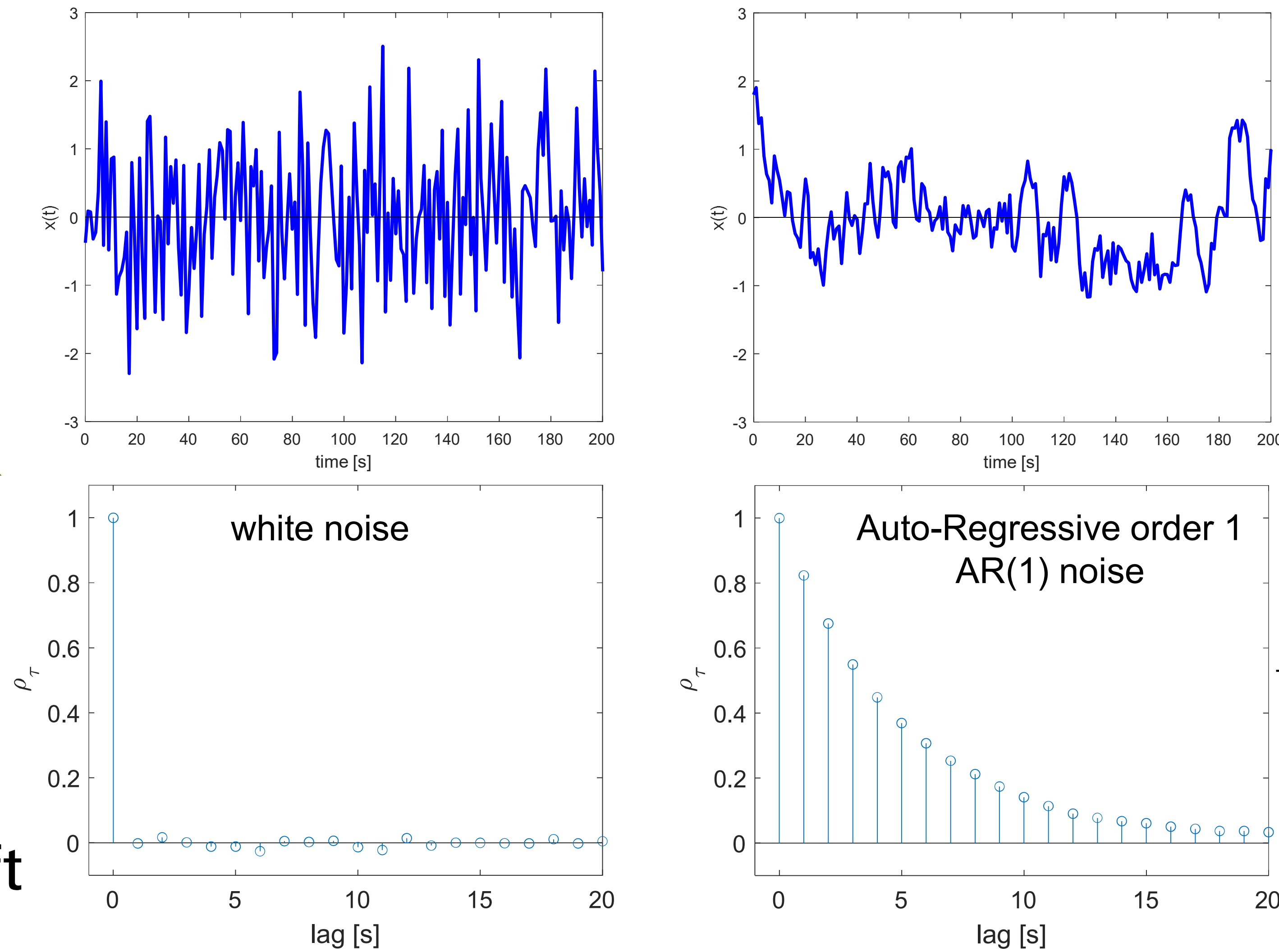
Noise: time correlation

interactive Python demo in MUDE textbook
simulated example section 6 Autoregressive process

time series

normalized autocovariance function (ACF)



after trend removal (functional effects), noise still shows a kind of memory ...

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Observation theory (week 1.7)

will guide us how to **detrend** the time series

Consider the linear model of observation equations as

$$Y = \mathrm{A}\mathrm{x} + \epsilon, \quad \mathbb{D}(Y) = \Sigma_Y$$

Recall that the BLUE of $\mathbf{x}$ is:

$$\hat{X} = (\mathrm{A}^T \Sigma_Y^{-1} \mathrm{A})^{-1} \mathrm{A}^T \Sigma_Y^{-1} Y, \quad \Sigma_{\hat{X}} = (\mathrm{A}^T \Sigma_Y^{-1} \mathrm{A})^{-1}$$

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Observation theory (week 1.7)

functional model: components of time series

- trend
- seasonality ← find frequency with PSD (week 2.3): $S(k\Delta f) = \frac{1}{T}|X_k|^2$ (periodogram)
- offset (jump/break)
- …

'peaks in the spectrum'

$$\overbrace{\begin{bmatrix} Y_1 \\ \vdots \\ Y_{k-1} \\ Y_k \\ \vdots \\ Y_m \end{bmatrix}}^{Y} = \overbrace{\begin{bmatrix} 1 & t_1 & \cos\omega_0 t_1 & \sin\omega_0 t_1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & t_{k-1} & \cos\omega_0 t_{k-1} & \sin\omega_0 t_{k-1} & 0 \\ 1 & t_k & \cos\omega_0 t_k & \sin\omega_0 t_k & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & t_m & \cos\omega_0 t_m & \sin\omega_0 t_m & 1 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} y_0 \\ r \\ a \\ b \\ o \end{bmatrix}}^{x} + \overbrace{\begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_{k-1} \\ \epsilon_k \\ \vdots \\ \epsilon_m \end{bmatrix}}^{\epsilon}$$

$$\Sigma_Y = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1m} \\ \sigma_{21} & \sigma_2^2 & & \\ \vdots & \vdots & \ddots & \\ \sigma_{m1} & \sigma_{m2} & \dots & \sigma_m^2 \end{bmatrix}$$

see section 4.3: Modelling and estimation

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Stochastic model – time series (week 2.4)

stochastic model: time correlation

MUDE textbook: Sections 4.4 – 4.8

and, next hour of lecture

- stationarity of time series (§4.4)
- auto-covariance function (§4.5)
- Auto Regressive process (§4.6)
- Autocorrelation and PSD (§4.7)
- forecasting (§4.8)

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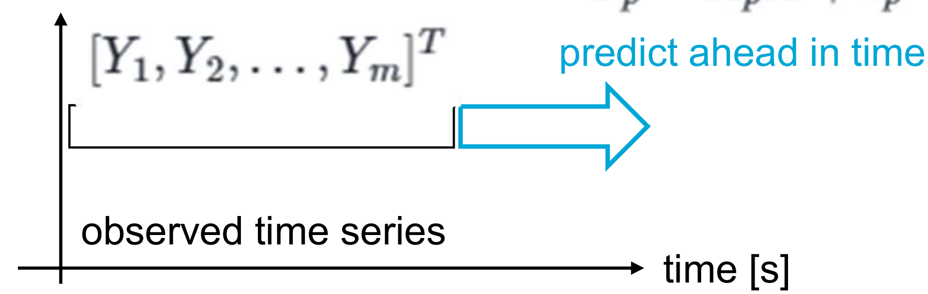
Purpose of time series analysis: forecasting

$$\hat{Y}_p = \mathrm{A}_p\hat{X} + \hat{\epsilon}_p$$



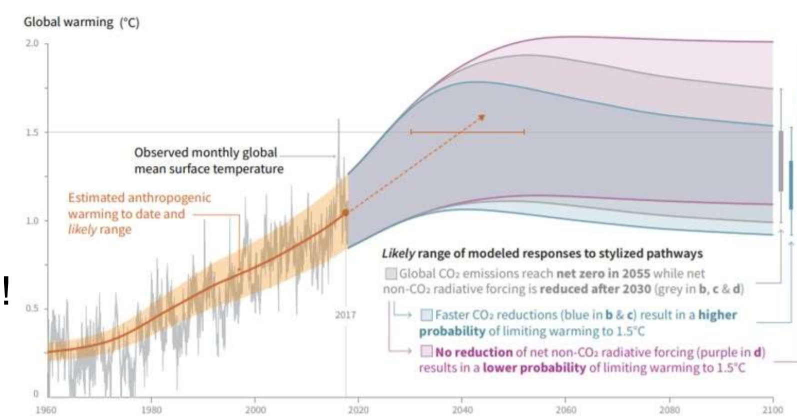
observable = signal + noise

predict both signal and noise (and, account for uncertainty)

part of noise process
is 'memory'

(real) random part

exploit 'memory'-part of noise to improve prediction!



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Five topics will be covered

1. Re-cap Observation Theory (week 1.7) (Section 3)
2. Stationarity of time series (Section 4)
3. Auto-covariance function (ACF; Section 5)
4. Auto Regressive process (Section 6)
5. Time series forecasting (Section 8)

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Application fields of TSA

1. Structural health monitoring (vibration analysis), life cycle management
2. Geo-engineering and geophysics (deformation, seismic)
3. Climate and meteorology (rainfall, temperature, pressure, wind speed)
4. Geoscience (GNSS, InSAR, tide, sea level rise)
5. Environmental engineering (water management, air pollution)
6. Traffic management (traffic flows, # of passengers / vehicles)
7. Econometrics and finance (stock prices, quarterly sales, interest rates)
8. ….

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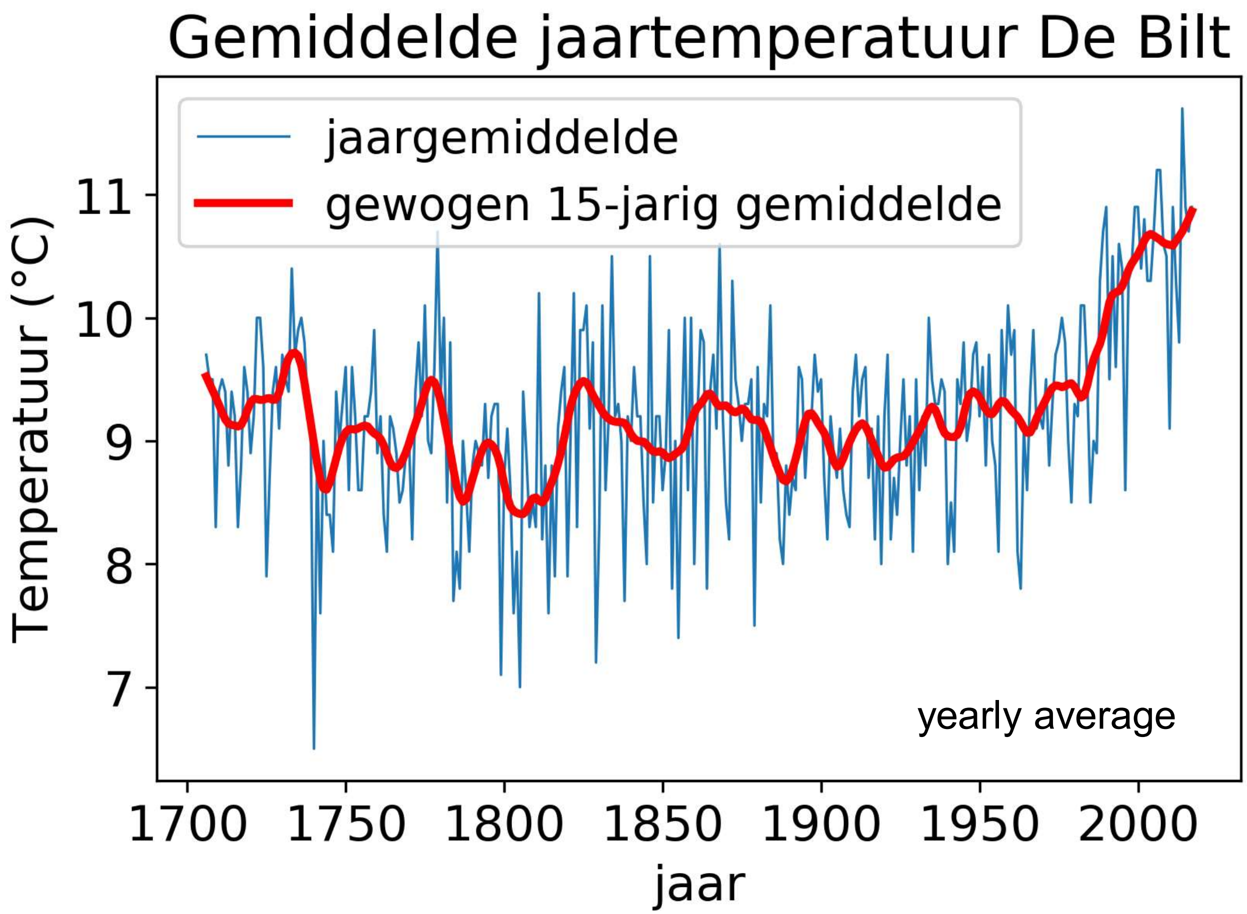
Examples of time series

Temperature time series



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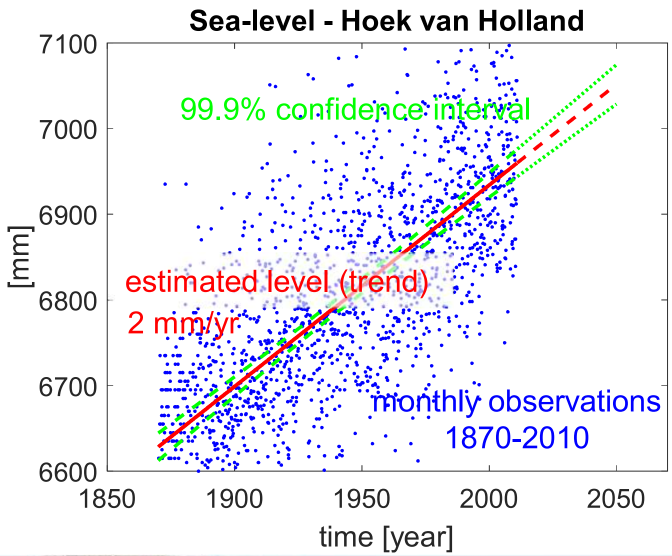
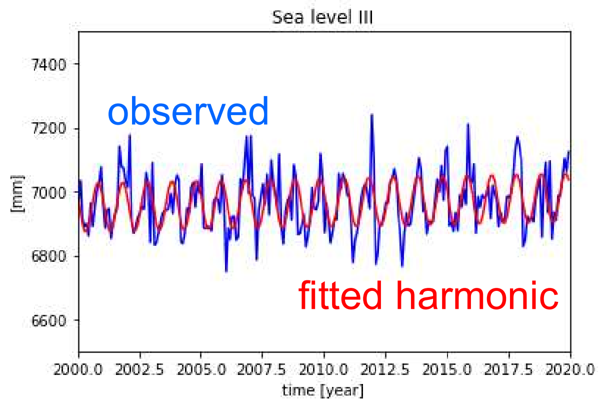
Sea level time series – tide gauge





https://www.tudelft.nl/en/2022/tu-delft/tu-delft-researchers-sea-level-rise-along-dutch-coastline-accelerating

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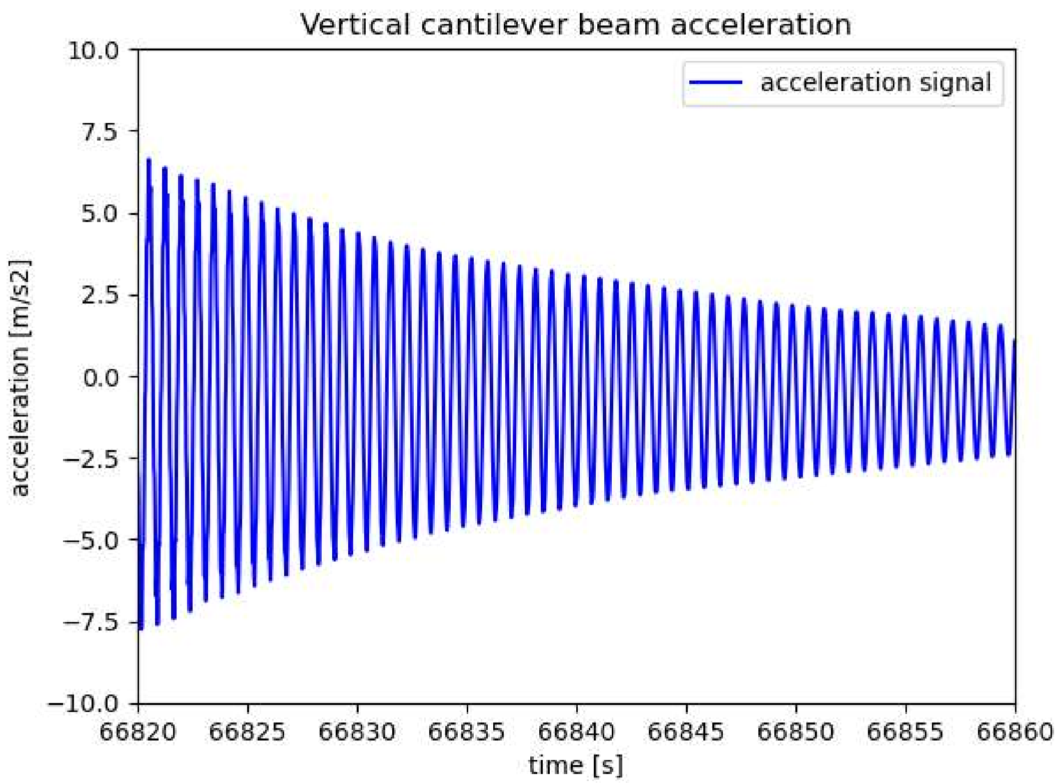
Cantilever beam - accelerometer



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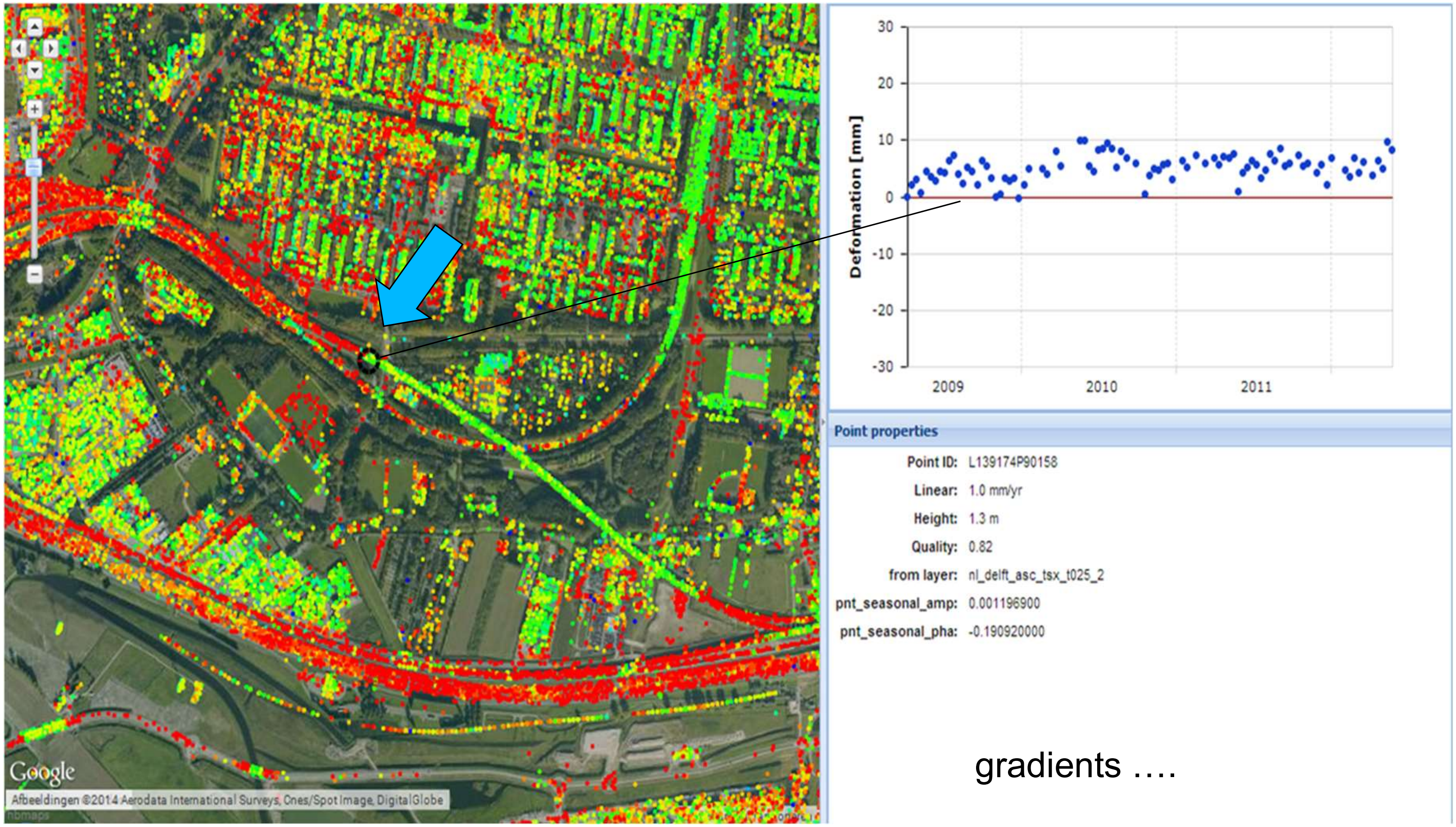
Structural health monitoring: InSAR deformation

infrastructure

Earth Observation:
ESA Sentinel 1
satellite



gradients ….

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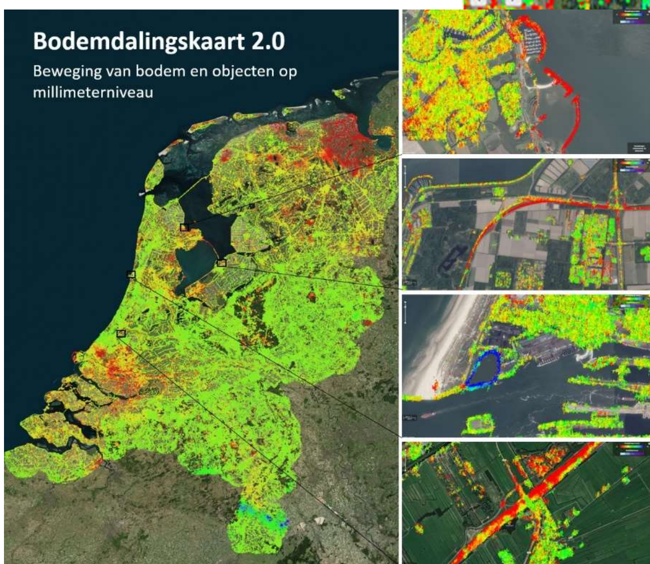
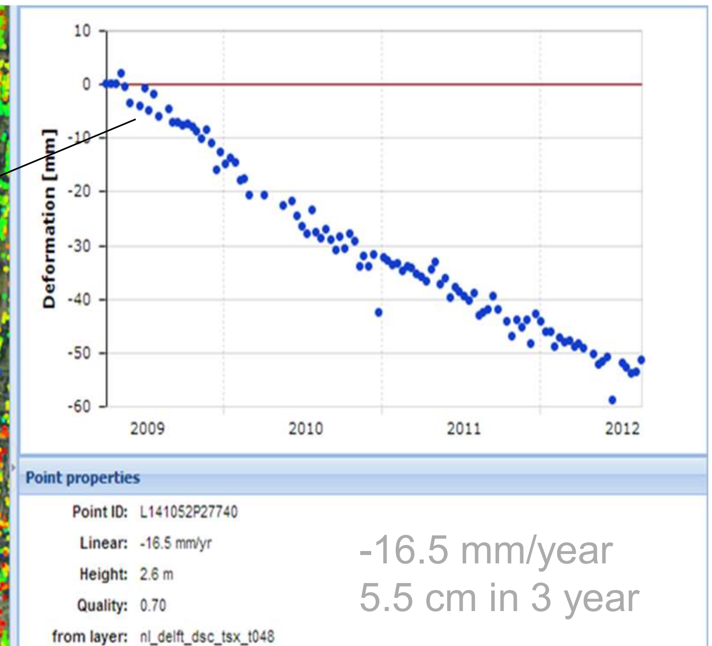
Structural health monitoring: InSAR deformation

infrastructure



https://bodemdalingskaart.nl/



-16.5 mm/year
5.5 cm in 3 year

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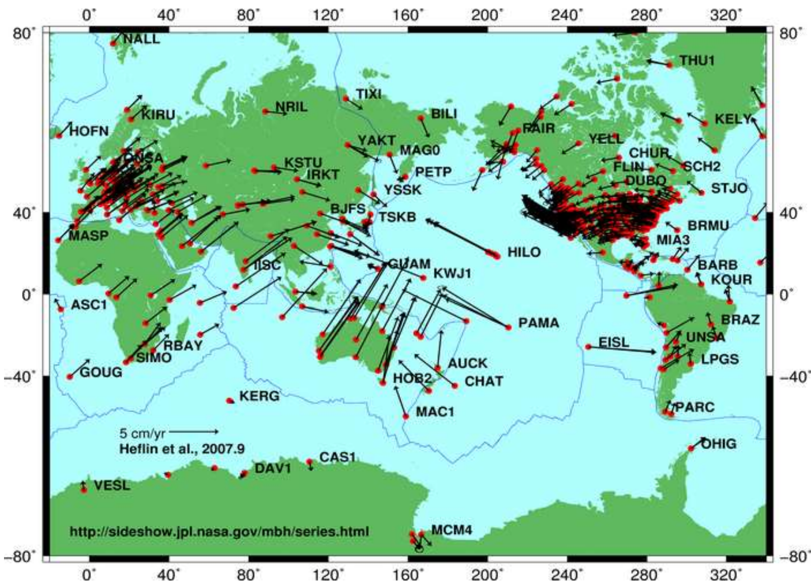
GNSS position time series

tectonic plate motion (Earthquakes …)

global velocities: IGS stations



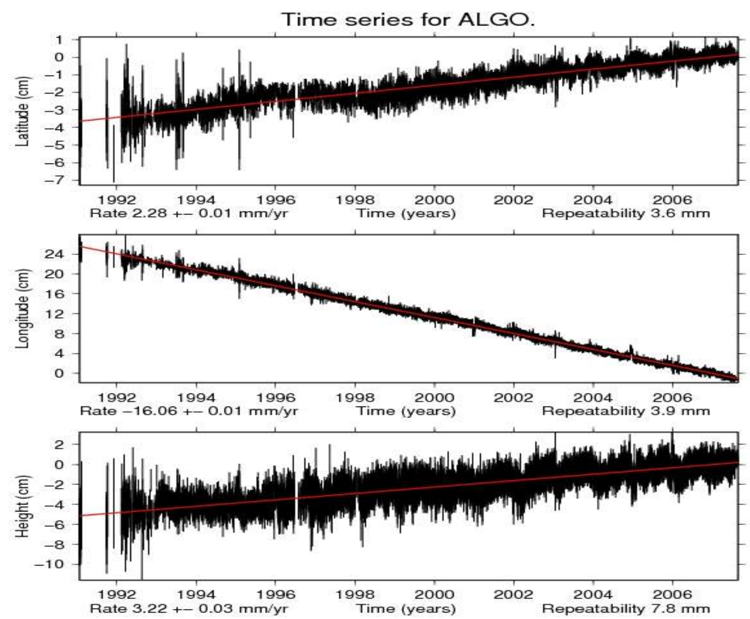
JOURNAL OF GEOPHYSICAL RESEARCH
Solid Earth
AN AGU JOURNAL

Geodesy and Gravity/Tectonophysics | Free Access

Assessment of noise in GPS coordinate time series: Methodology and results

A. R. Amiri-Simkooei, C. C. J. M. Tiberius, P. J. G. Teunissen

ALGO station in Canada



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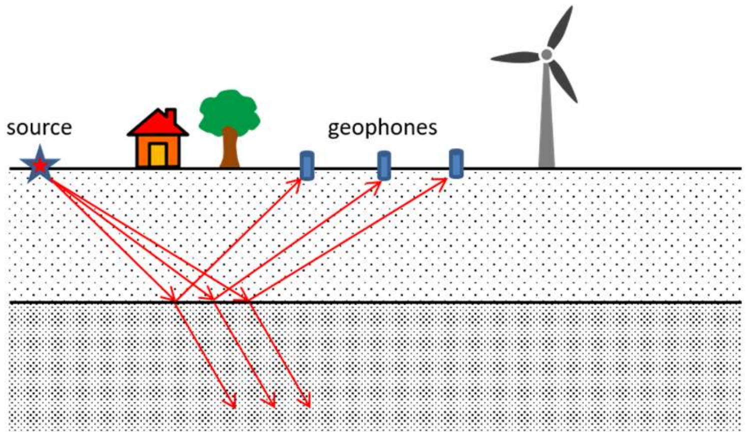
Seismic reflection



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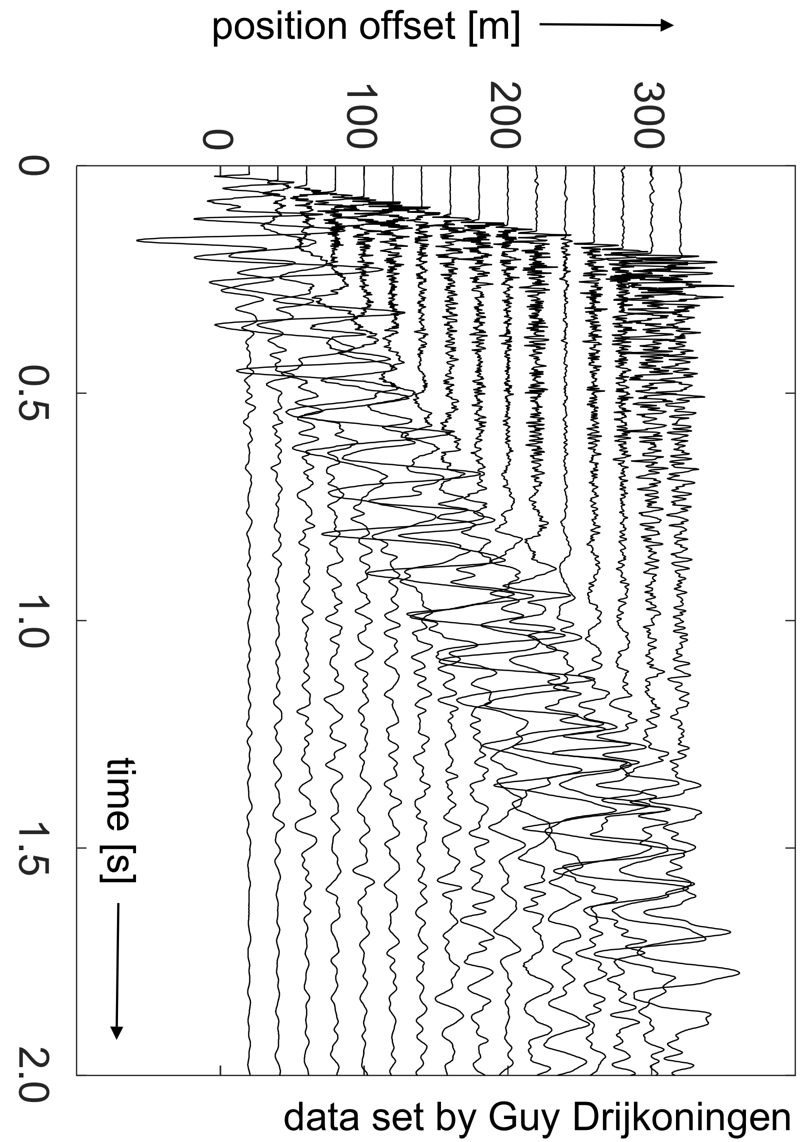
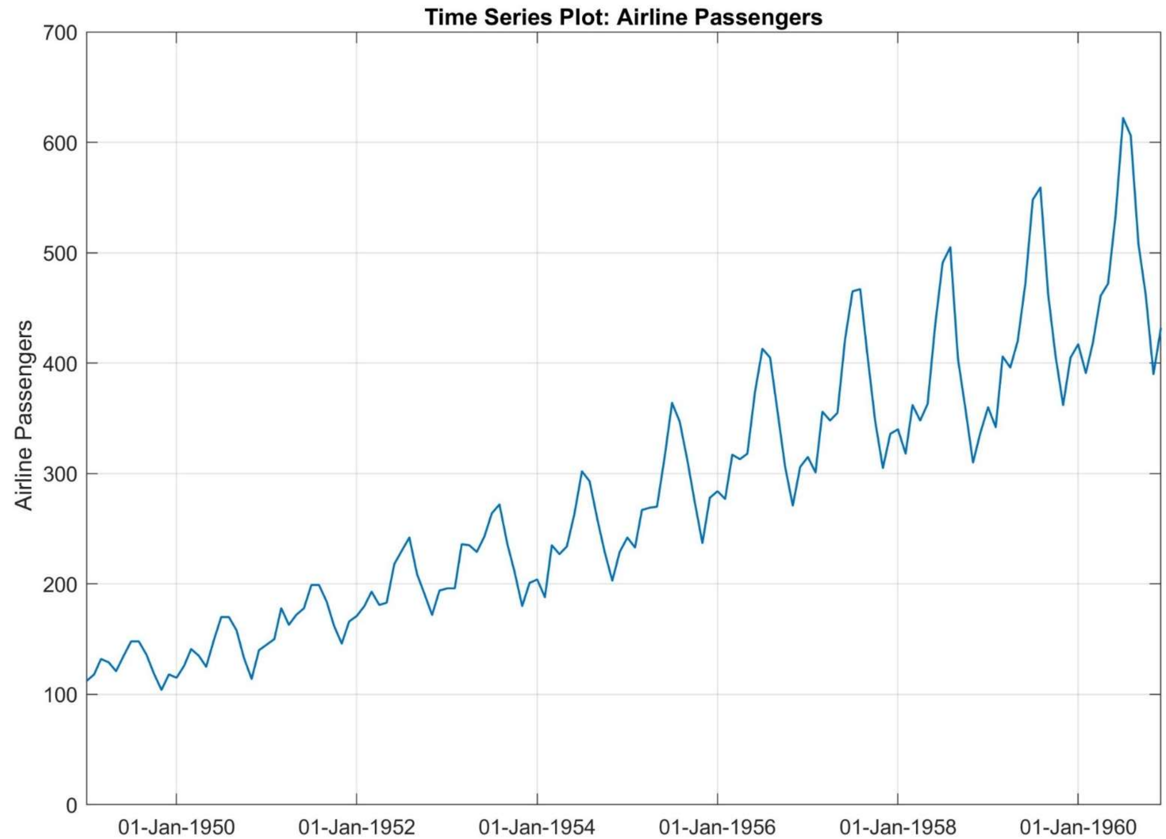
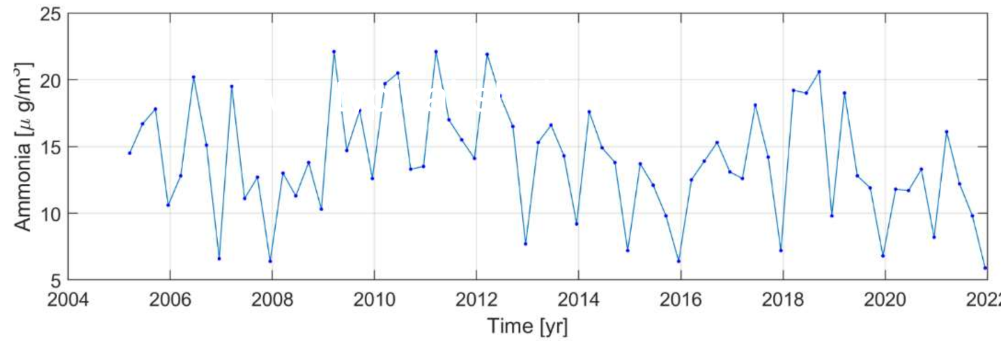
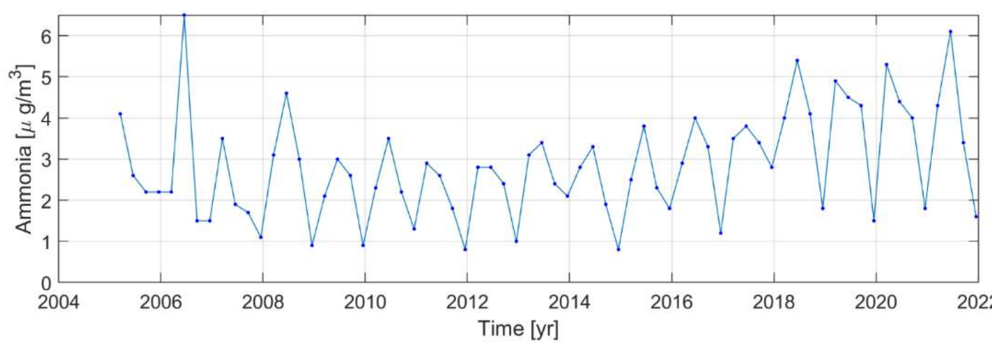


Monthly air passengers (1949-1960)



Concentration NH_3 ammonia in nature areas





Monitoring Network (MAN): 331 positions

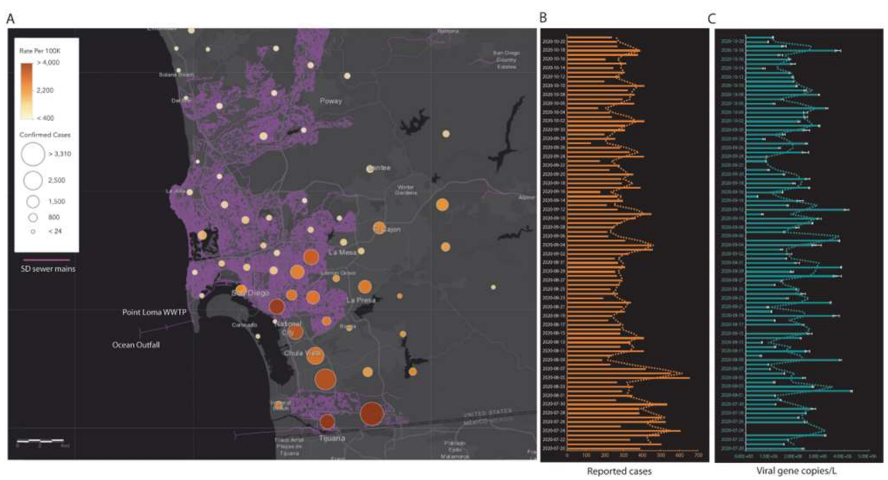
Forecasting Covid-19 cases from wastewater monitoring

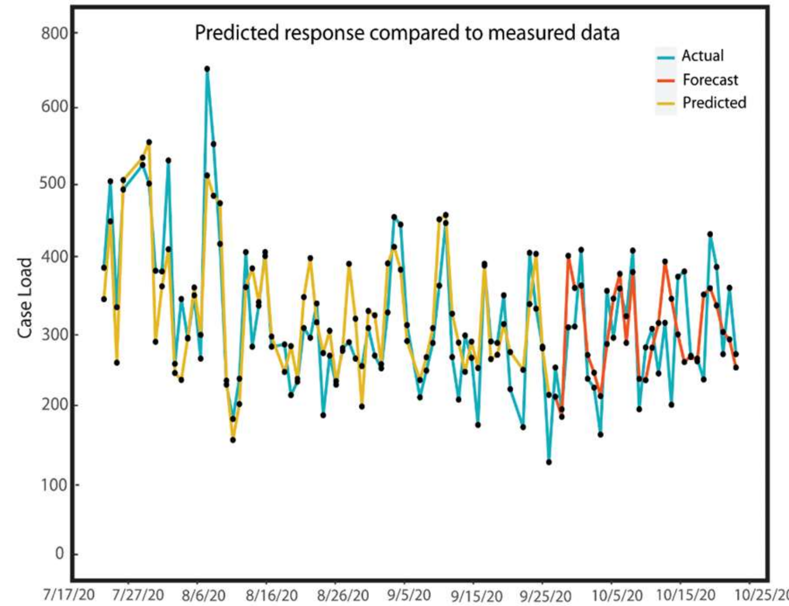
| Environmental Microbiology | Observation | 2 March 2021

High-Throughput Wastewater SARS-CoV-2 Detection Enables Forecasting of Community Infection Dynamics in San Diego County

Authors: Smruthi Karthikeyan, Nancy Ronquillo, Pedro Belda-Ferre, Destiny Alvarado, Tara Javidi, Christopher A. Longhurst, Rob Knight | AUTHORS INFO & AFFILIATIONS

DOI: https://doi.org/10.1128/mSystems.00045-21 • Check for updates





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Groundwater Time Series Analysis with Pastas

groundwater NGWA

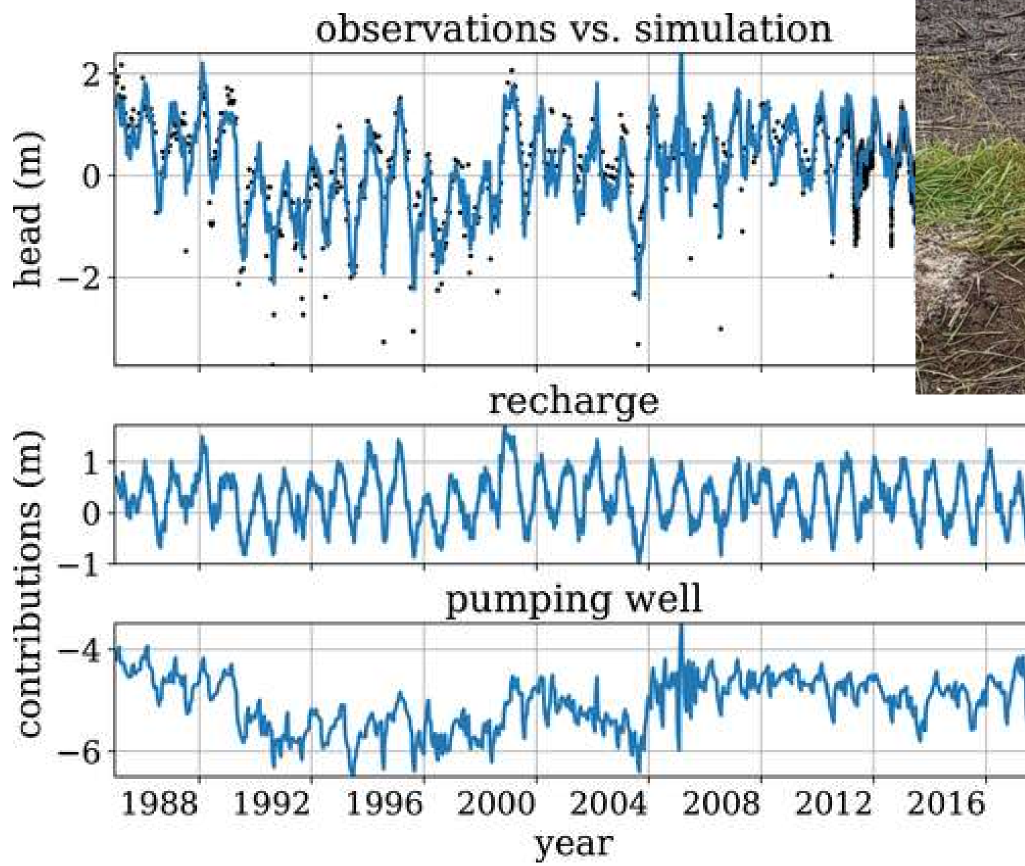
Methods Notes/ | Open Access

Pastas: Open Source Software for the Analysis of Groundwater Time Series

by Raoul A. Collenteur, Mark Bakker, Ruben Caljé, Stijn A. Klop, Frans Schaars

PASTAS

http://github.com/pastas/pastas



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Conclusion

Time series analysis has many applications in different fields of civil, environmental and geoscience engineering. The subject is closely linked to those Observation Theory (MUDE Q1) and Signal Processing (week 2.3).

Week 2.4: basics of time series analysis. More advanced TSA include:

- Dynamic time series analysis
- Multivariate time series analysis
- Noise assessment in time series analysis
- Data-driven time series analysis (e.g. machine learning)

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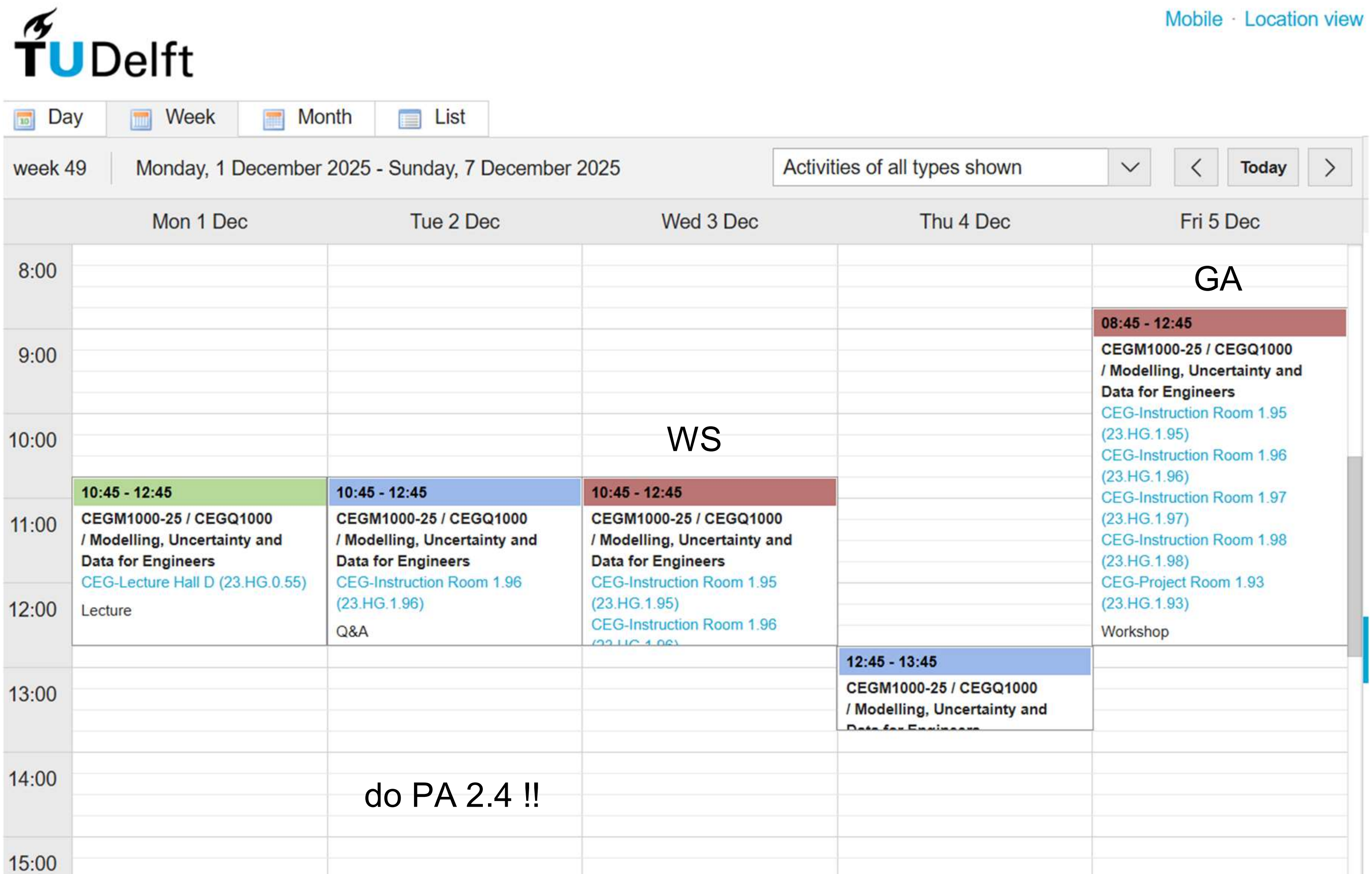
Time Series Analysis – week 2.4



Time series analysis

Lecture

Week 2.4, 1 Dec. 2025

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Modelling, Uncertainty and Data for Engineers

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Two aspects on time series analysis (TSA)

Two main goals for TSA:

- To explain past and present state of TS
  - Identifying nature of phenomenon represented by time series data to study long-term trend, seasonality and noise process of time series.
- To use past data for predicting future values (events)
  - Prediction (or forecasting) uses past observed values of time series, try to model, and hence predict future time series values. Think of forecasting sales of a particular product, forecasting of stock price, or weather forecasting.

Stationary time series (Section 4)

- statistical properties do not depend on time (at which it is observed)

i.e. parameters such as mean and (co)variance of time series should remain constant over time

How to stationarize time series?

- detrending → least-squares fit / Best Linear Unbiased Estimation (BLUE)

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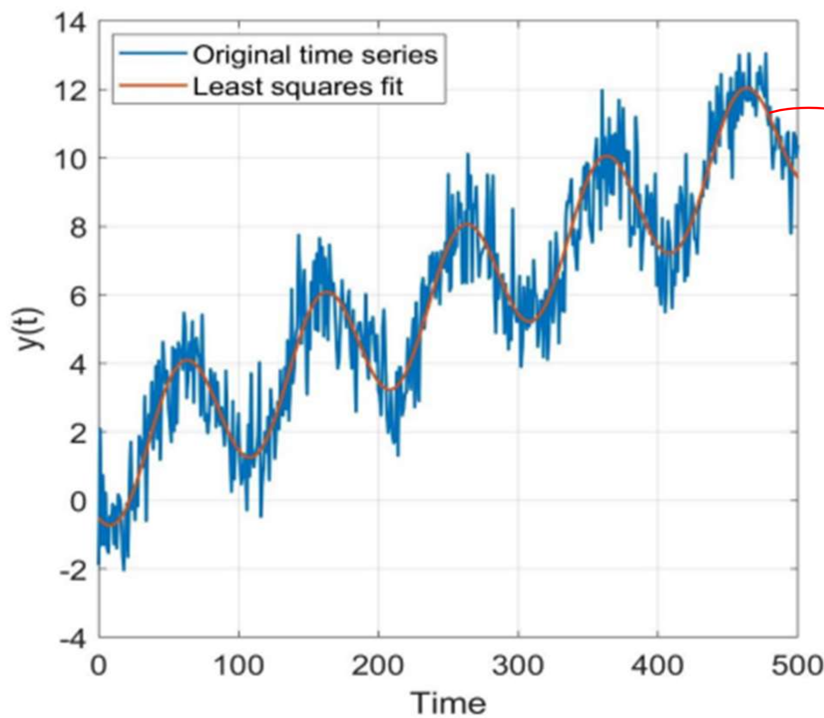
Stationary time series: example

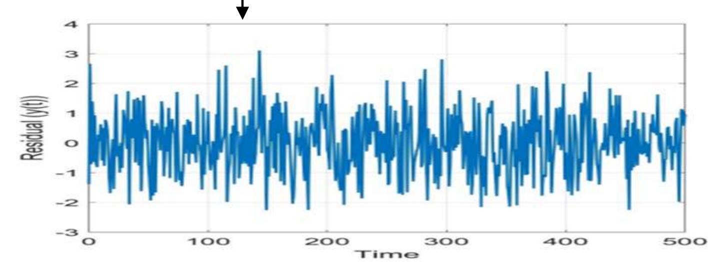
linear (intercept & slope), and seasonal trend (cos & sin), and noise



remove trend
only noise left

$$Y = A\hat{X} + \hat{\epsilon}$$

residuals



$$S := \hat{\epsilon} = Y - A\hat{X}$$

Autocovariance function (ACF) (Section 5): formal / theoretical

The *formal* (or: theoretical) autocovariance is defined as

mean $\mathbb{E}(S) = \mu$

$$Cov(S_{t+\tau}, S_t) = \underbrace{\mathbb{E}(S_{t+\tau}S_t)}_{\text{autocorrelation}} - \mu^2 = c_\tau$$

stationary time series, $S = [S_1, S_2, \ldots, S_m]^T$.

$$Cov(S_{t+\tau}, S_t) = Cov(S_t, S_{t-\tau})$$

for zero mean: autocovariance = autocorrelation

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Empirical autocovariance function (ACF)

For a given stationary time series $S = [S_1, S_2, \ldots, S_m]^T$, the least-squares estimator of the **autocovariance function** is given by

$$\hat{C}_\tau = \frac{1}{m-\tau} \sum_{i=1}^{m-\tau} (S_{i+\tau} - \mu)(S_i - \mu), \qquad \tau = 0, 1, \ldots, m-1$$

The least-squares estimator of **autocorrelation** (also called empirical autocorrelation function) is then

$$\hat{R}_\tau = \frac{1}{m-\tau} \sum_{i=1}^{m-\tau} S_{i+\tau} S_i, \qquad \tau = 0, 1, \ldots, m-1$$

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Empirical autocovariance: example

zero mean, m=5

$\hat{C}_\tau = \hat{R}_\tau$

$\tau = 0$

t	0	1	2	3	4
$s_{t+\tau}$	2	1	0	−1	−2
s_t	2	1	0	−1	−2
$s_{t+\tau}s_t$	4	1	0	1	4
$\hat{r}_{\tau=0}$	10/5				

sum, and divide by overlap

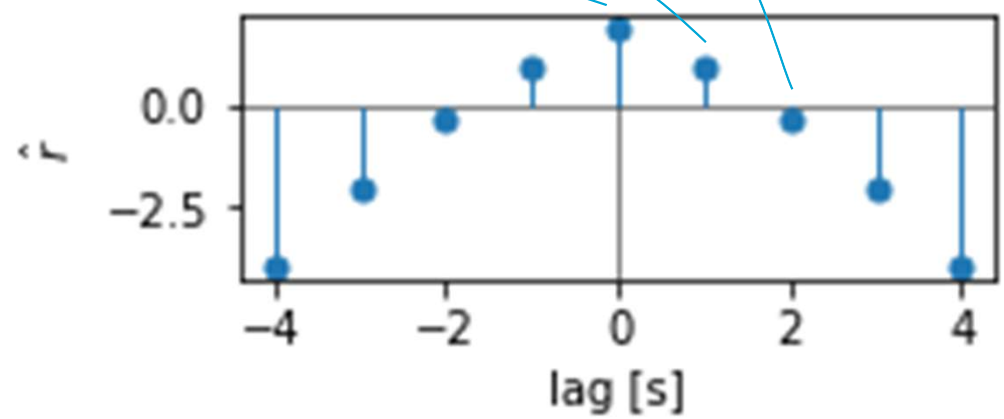
$\tau = 1$

t		0	1	2	3	4
$s_{t+\tau}$	2	1	0	−1	−2	
s_t		2	1	0	−1	−2
$s_{t+\tau}s_t$		2	0	0	2	
$\hat{r}_{\tau=1}$	4/4					

$\tau = 2$

t			0	1	2	3	4
$s_{t+\tau}$	2	1	0	−1	−2		
s_t			2	1	0	−1	−2
$s_{t+\tau}s_t$			0	−1	0		
$\hat{r}_{\tau=2}$	−1/3						

etc for $\tau = 3$ and $\tau = 4$, and for negative lags



Auto-Regressive process (AR) – Section 6

$$S_t = \overbrace{\phi_1 S_{t-1} + \ldots + \phi_p S_{t-p}}^{\text{AR process}} + e_t$$

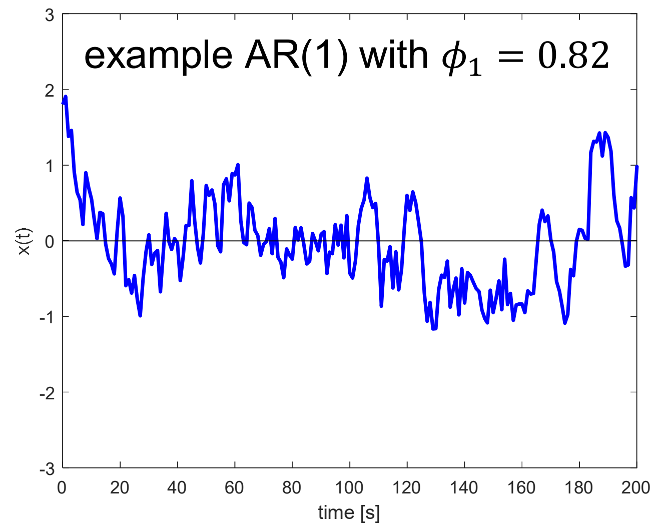
linear combination of past values,
plus (new) random error

$$S_t = \sum_{i=1}^{p} \phi_i S_{t-i} + e_t$$

AR order *p*

purely random
(white noise)

$$\mathbb{E}(S_t) = 0, \quad \mathbb{D}(S_t) = \sigma^2, \quad \forall t$$



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first order Auto-Regressive process – AR(1)

$$\mathbb{E}(S) = \mathbb{E}\begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \mathbb{D}(S) = \Sigma_S = \sigma^2 \begin{bmatrix} 1 & \phi & \dots & \phi^{m-1} \\ \phi & 1 & \dots & \phi^{m-2} \\ \vdots & \vdots & \ddots & \vdots \\ \phi^{m-1} & \phi^{m-2} & \dots & 1 \end{bmatrix}$$

ϕ larger → longer 'memory'

$|\phi| < 1$ → stationary

(formal) normalized autocovariance $\rho_\tau = \frac{Cov(S_{t+\tau} S_t)}{Cov(S_t S_t)}$

$$\rho_{\tau=1} = \frac{\phi\sigma^2}{\sigma^2} = \phi$$

Find parameter ϕ

Example: Parameter estimation of AR(1)

The AR(1) process is of the form

$$S_t = \phi_1 S_{t-1} + e_t$$

In order to estimate the ϕ_i we can set up the following linear model of observation equations (starting from $t = 2$):

$$\begin{bmatrix} S_2 \\ S_3 \\ \vdots \\ S_m \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_{m-1} \end{bmatrix} [\phi_1] + \begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_m \end{bmatrix}$$

The BLUE estimator of ϕ is given by:

$$\hat{\phi} = (\mathrm{A}^T\mathrm{A})^{-1}\mathrm{A}^T S$$

Where $\mathrm{A} = [S_1 \quad S_2 \quad \cdots \quad S_{m-1}]^T$ and $S = [S_2 \quad S_3 \quad \cdots \quad S_m]^T$.

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Forecasting – section 8

(observed) time series: $Y(t) = [Y(t_1), Y(t_2), \ldots, Y(t_m)]^T$

1. Estimate the signal-of-interest $\hat{X} = (\mathrm{A}^T \Sigma_Y^{-1} \mathrm{A})^{-1} \mathrm{A}^T \Sigma_Y^{-1} Y$.
2. Model the noise using the Autoregressive (AR) model, using the stationary time series $S := \hat{\epsilon} = Y - \mathrm{A}\hat{X}$.
3. Predict the signal-of-interest: $\hat{Y}_{signal} = \mathrm{A}_p \hat{X}$, where A_p is the design matrix describing the functional relationship between the future values Y_p and x.
4. Predict the noise $\hat{\epsilon}_p$ based on the AR model. $\hat{\epsilon}_p = \Sigma_{Y_p Y} \Sigma_Y^{-1} \hat{\epsilon}$, where $\Sigma_{Y_p Y}$ is the covariance matrix between the future values Y_p and the observed values Y.
5. Predict future values of the time series: $\hat{Y}_p = \mathrm{A}_p \hat{X} + \hat{\epsilon}_p$.

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Forecasting

4. Predict the noise $\hat{\epsilon}_p$ based on the AR model. $\hat{\epsilon}_p = \Sigma_{Y_pY}\Sigma_Y^{-1}\hat{\epsilon}$, where Σ_{Y_pY} is the covariance matrix between the future values Y_p and the observed values Y.

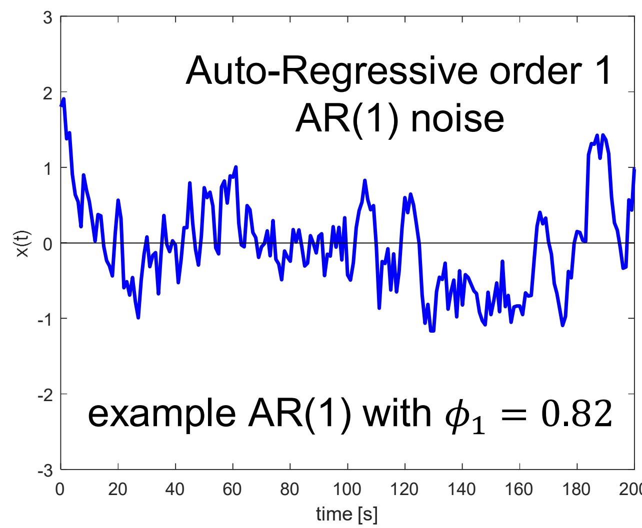
For AR(1) this implies simply: $S_t = \phi S_{t-1} + e_t$ with $S := \hat{\epsilon} = Y - A\hat{X}$

(one step ahead)

$\hat{\epsilon}_p$ $\hat{\epsilon}$

purely random (white noise), zero mean

(hence, best prediction we can think of: $\hat{e}_t = 0$)



Best linear unbiased prediction (BLUP) - optional

Consider the (augmented) linear model of observation equations as

$$\begin{bmatrix} Y \\ Y_p \end{bmatrix} = \begin{bmatrix} A \\ A_p \end{bmatrix} x + \begin{bmatrix} \epsilon \\ \epsilon_p \end{bmatrix}, \qquad \mathrm{D}\begin{pmatrix} Y \\ Y_p \end{pmatrix} = \begin{bmatrix} \Sigma_Y & \Sigma_{YY_p} \\ \Sigma_{Y_pY} & \Sigma_{Y_p} \end{bmatrix}$$

The best linear unbiased estimation (BLUE) of x is

$$\hat{X} = (A^T \Sigma_Y^{-1} A)^{-1} A^T \Sigma_Y^{-1} Y$$

The 'best linear unbiased prediction' (BLUP) of Y_p is (proof is not provided)

$$\hat{Y}_p = A_p \hat{X} + \Sigma_{Y_pY} \Sigma_Y^{-1} (Y - A\hat{X})$$

With the covariance matrix

$$\Sigma_{\hat{Y}_p} = A_p \Sigma_{\hat{X}} A_p^T + \Sigma_{Y_pY} \Sigma_Y^{-1} \Sigma_{\hat{\epsilon}} \Sigma_Y^{-1} \Sigma_{YY_p}$$

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