Modelling, Uncertainty and Data for Engineers (MUDE)

Optimization Week. Plenary Lecture

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Lecturers and Beyond

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A team you will soon meet!

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Lecture Outline

Part I

1. Optimization Origin
2. General Components
3. Optimization vs. Simulation

Part II

1. Optimization Examples
2. Classification

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Learning Objectives

By the end of the first part of this lecture, you should be able to:

Describe how optimization emerged from early mathematical, engineering, and operations research practices

Formulate a simple problem by specifying its objective, decision variables, parameters, and constraints

Compare optimization and simulation by distinguishing their roles, assumptions, and outputs when modelling complex real-world systems

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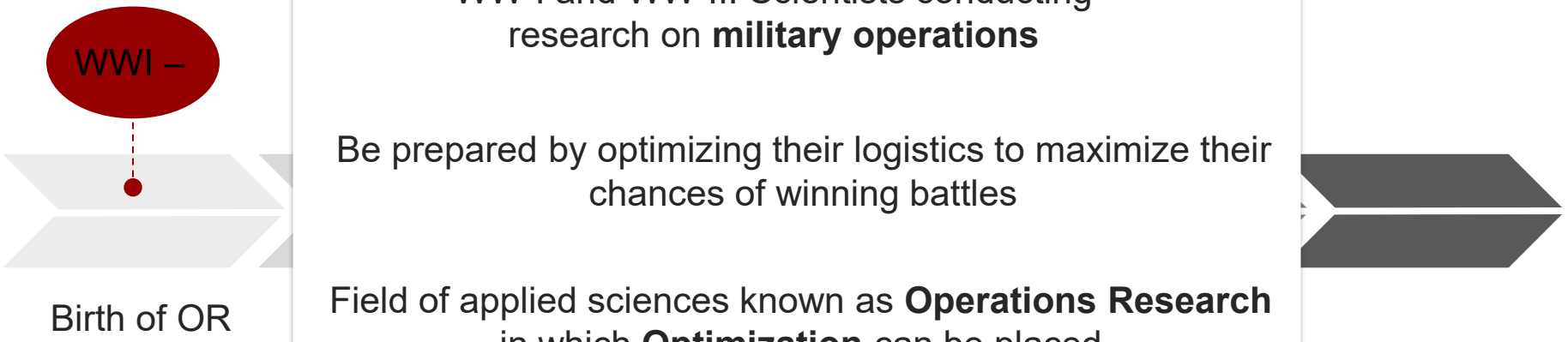
History Behind



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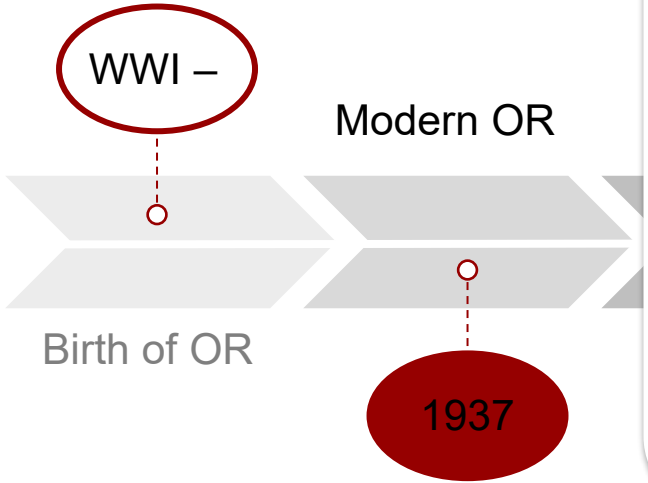

Modern OR: Initiative of A. P. Rowe

UK's early warning radar system, Chain Home (CH)

- How many radars do we need?
- Where should they be located?

Analysis expanded to include the operating personnel's behavior to plan HR of the system

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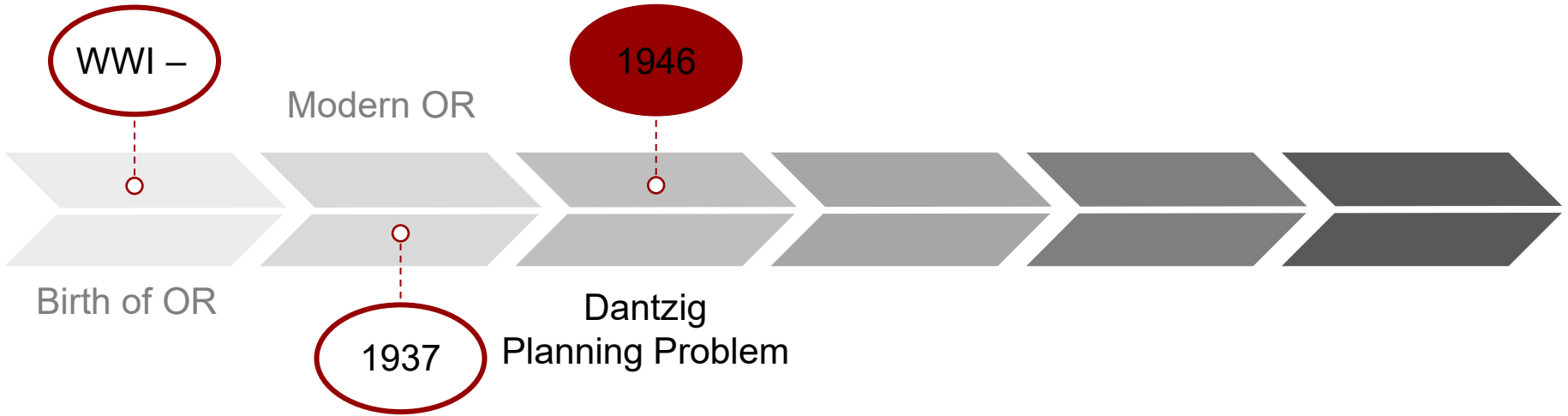
History Behind



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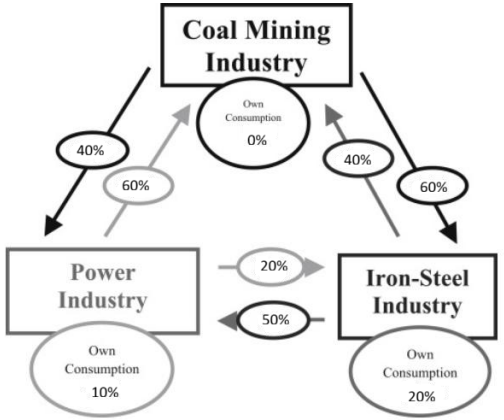

Input/Output matrix in a simple economy:

	Coal	Electricity	Steel
Coal	0	0.4	0.6
Electricity	0.6	0.1	0.2
Steel	0.4	0.5	0.2
	1	1	1

$$p_C = 0 \times p_C + 0.4 \times p_E + 0.6 \times p_S$$
$$p_E = 0.6 \times p_C + 0.1 \times p_E + 0.2 \times p_S$$
$$p_S = 0.4 \times p_C + 0.5 \times p_E + 0.2 \times p_S$$

Leontief In/Out Matrix

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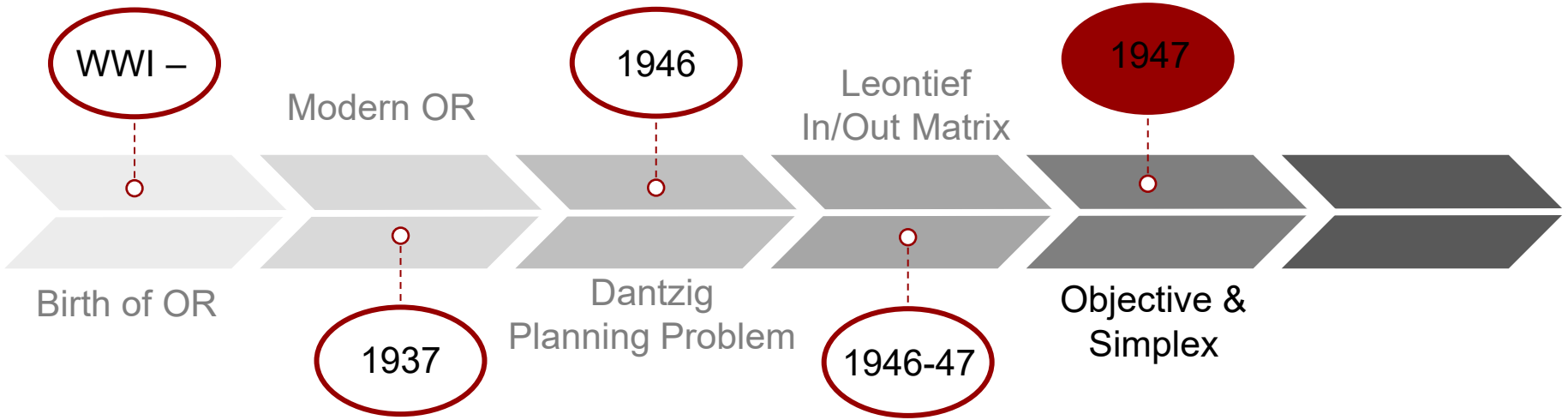

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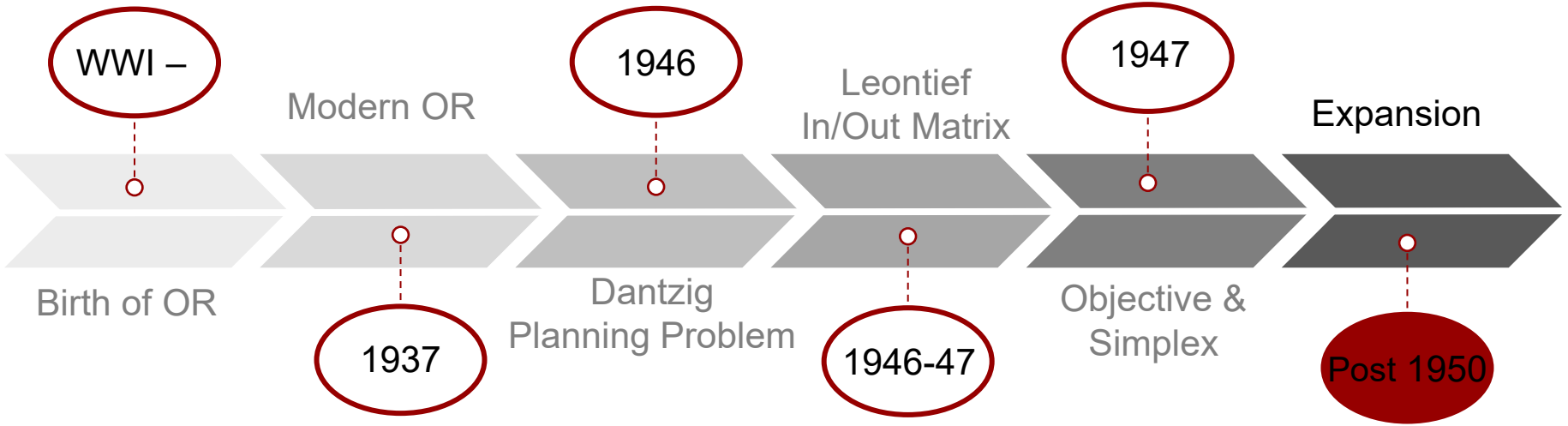

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Linear Programming

A chemical plant produces two liquid products, Product A and B, measured in tons per week.

The profit is €2k per ton of A and €1k per ton of B.

Each ton of A uses 3 hours of processing time in a key reactor, and B uses 1 hour. The reactor is available for max 150 hours per week.

Due to storage limitations, the total output of both products cannot exceed 90 tons.

To maintain market share, the plant must produce at least 40 tons of A.

The plant has a fixed contract to deliver exactly 20 tons of B every week

$$\max(F) = 2x + y$$

Objective function

$$3x + y \leq 150$$
$$x + y \leq 90$$
$$x \geq 40$$
$$y = 20$$

Linear inequalities/equalities

$$x, y \geq 0$$

Variables domain

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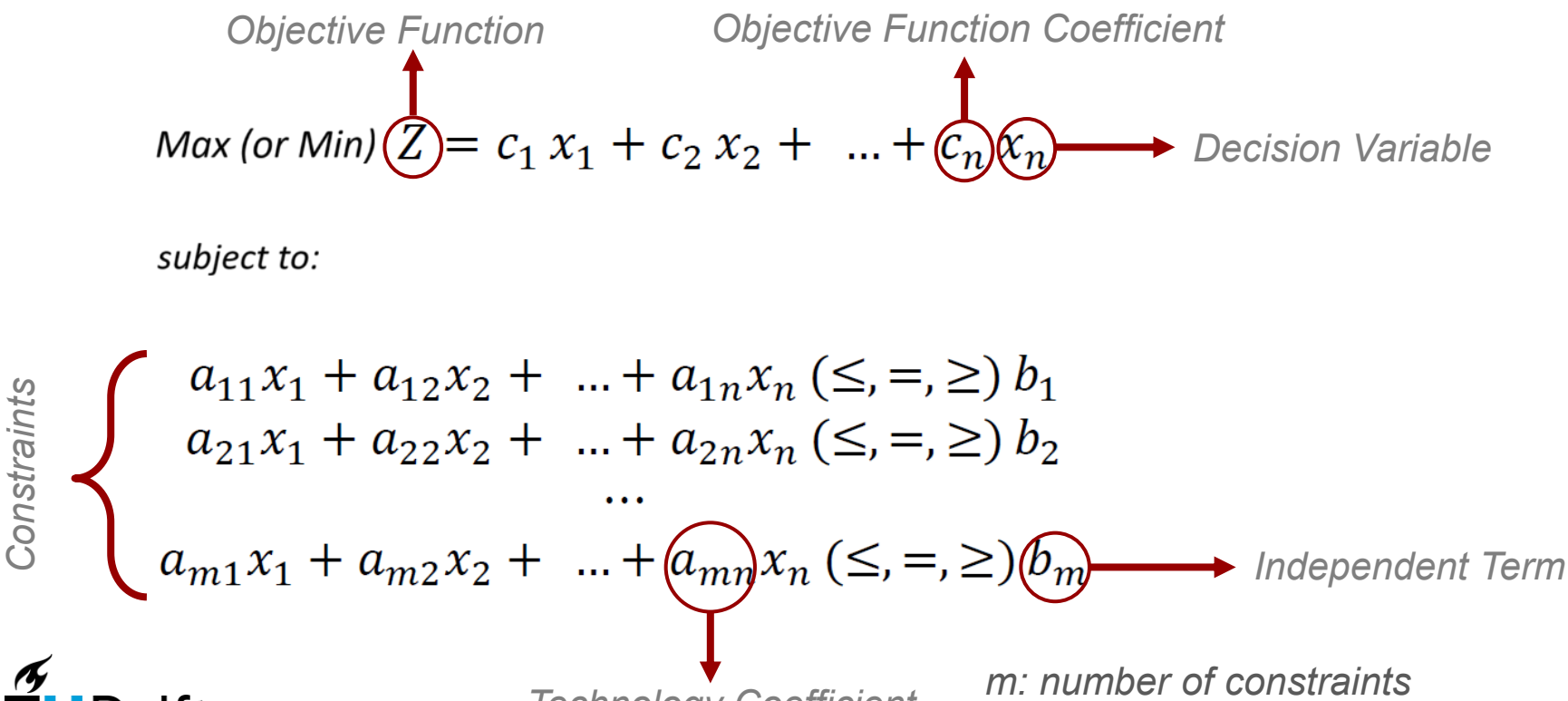
Linear Programming: General Notation



Some Questions

Select the correct answer, about the following program:

$$Max\ 5x_1 + 3x_2$$
$$s.t.$$
$$2x_1 + x_2 \geq 12$$
$$x_1 + 4x_2 \leq 20$$
$$x_1, x_2 \geq 0$$

- The problem is not a linear program because the objective contains two variables.
- There is no feasible region for this problem.
- ✓ Increasing the coefficient of x_1 in the objective may change the optimal solution.

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Some Questions

Which of the following objective functions are linear?

- ✓ $Max\ 4x_1 + 3x_2 - 2x_3$
- $Min\ 2x_1^2 + 5x_2$
- $Max\ 7x_1 - 3x_2x_3$
- ✓ $Max\ 9x_1 + (\log 10)\ x_2$

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Some Questions

A group of students are planning how to allocate their study hours between two courses.

Hours spent on Course A improves their grade by 3 points/hour, and on Course B, by 2 point/hour.

The students can study at most 50 hours per week.

They must spend at least 10 hours on Course A and 5 hours on course B.

$$Max\ 3x_1 + 2x_2$$

s.t.

$$x_1 + x_2 \leq 50$$

$$x_1 \geq 10$$

$$x_2 \geq 5$$

$$x_1, x_2 \geq 0$$

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Simulation versus Optimization

Optimization

Optimization searches through all feasible decisions to find the (near) best one according to an objective.

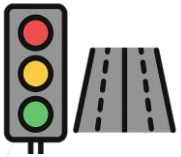
Optimization helps us **choose the best action**, not just predict outcomes.

Simulation

Simulation evaluates how a system behaves under a given set of inputs or decisions.

Simulation helps us **understand consequences**, not necessarily find optimal decisions.

Simulation for Optimal Solution



Possible?

A city wants to choose the best green–red split for a new intersection.

Engineers propose 8 candidate timing plans (e.g., 30–30, 40–20, adaptive versions, etc.).

For each plan, you can run a microsimulation (SUMO, etc) and measure: average delay, queue length, fuel consumption, emissions.

Since there are only **8 alternatives**, you can **simulate all of them**, compare results, and pick the best.

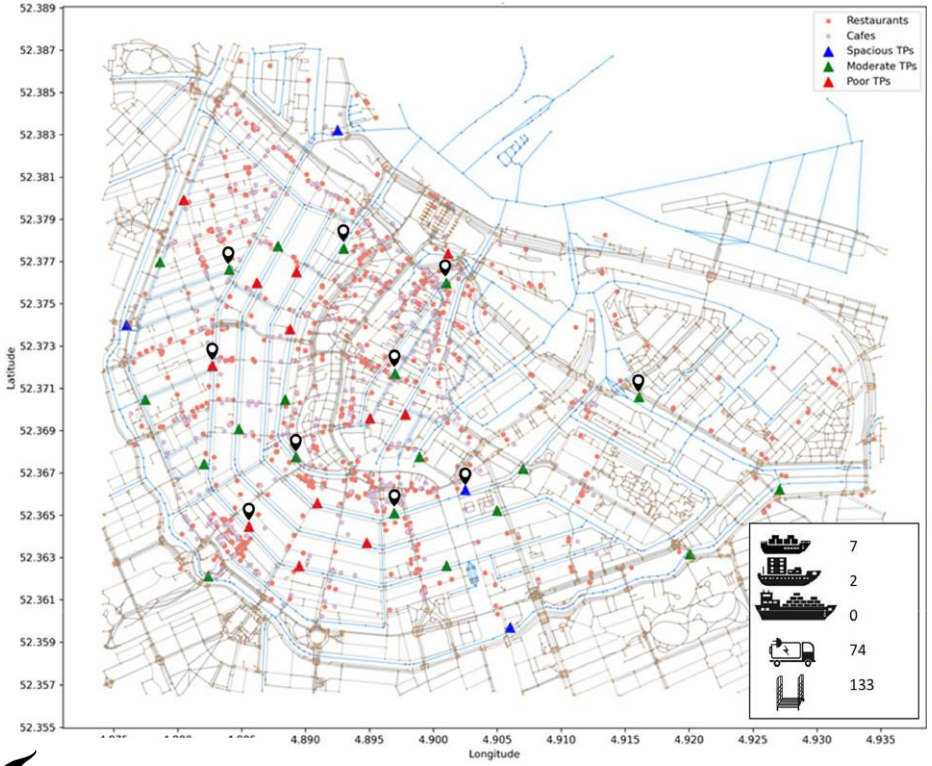
Possible?

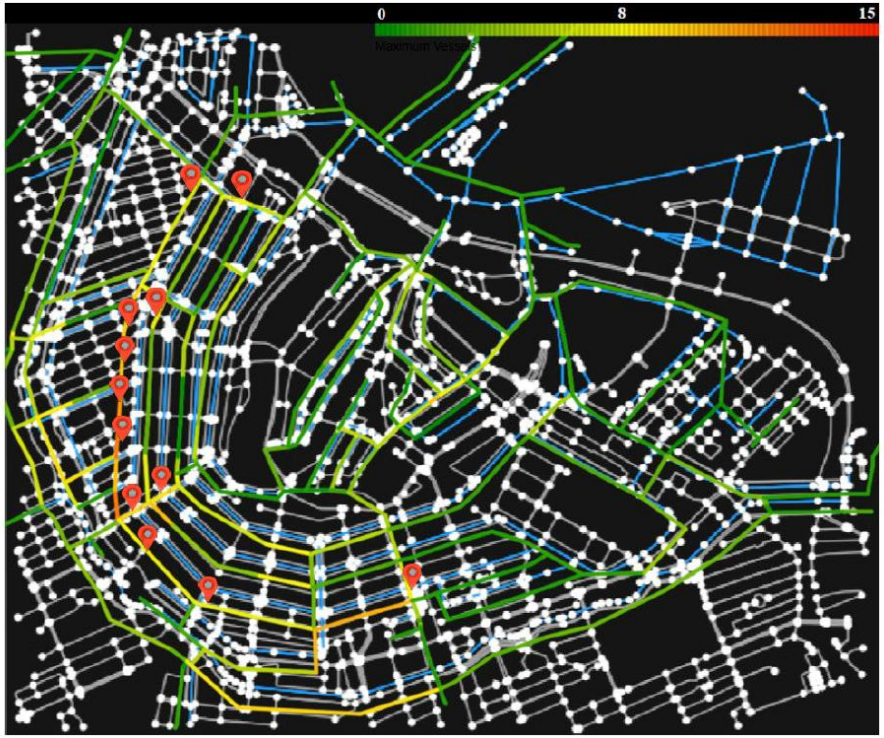
A warehouse wants to determine optimal set of **routes** taken by n autonomous robots that each visit a set of shelves and return to depot.

Each robot must visit **10 shelves**. The order in which a robot visits those shelves affects travel time and congestion.
For each robot, the number of possible route permutations is $10! = 3{,}628{,}800$.

If there are only **5 robots**, the total number of combined route sets is $(10!)^5$

Simulation & Optimization: Hand in Hand





Some Questions

You want to find the best design for a system. In which situation can simulation not be used to reliably find the optimal solution?

- Testing 10 versions of a traffic-signal timing plan for 2 intersections
- Evaluating 200 predefined bicycle network designs
- ✓ Designing a delivery route where 20 customers can be visited in any order

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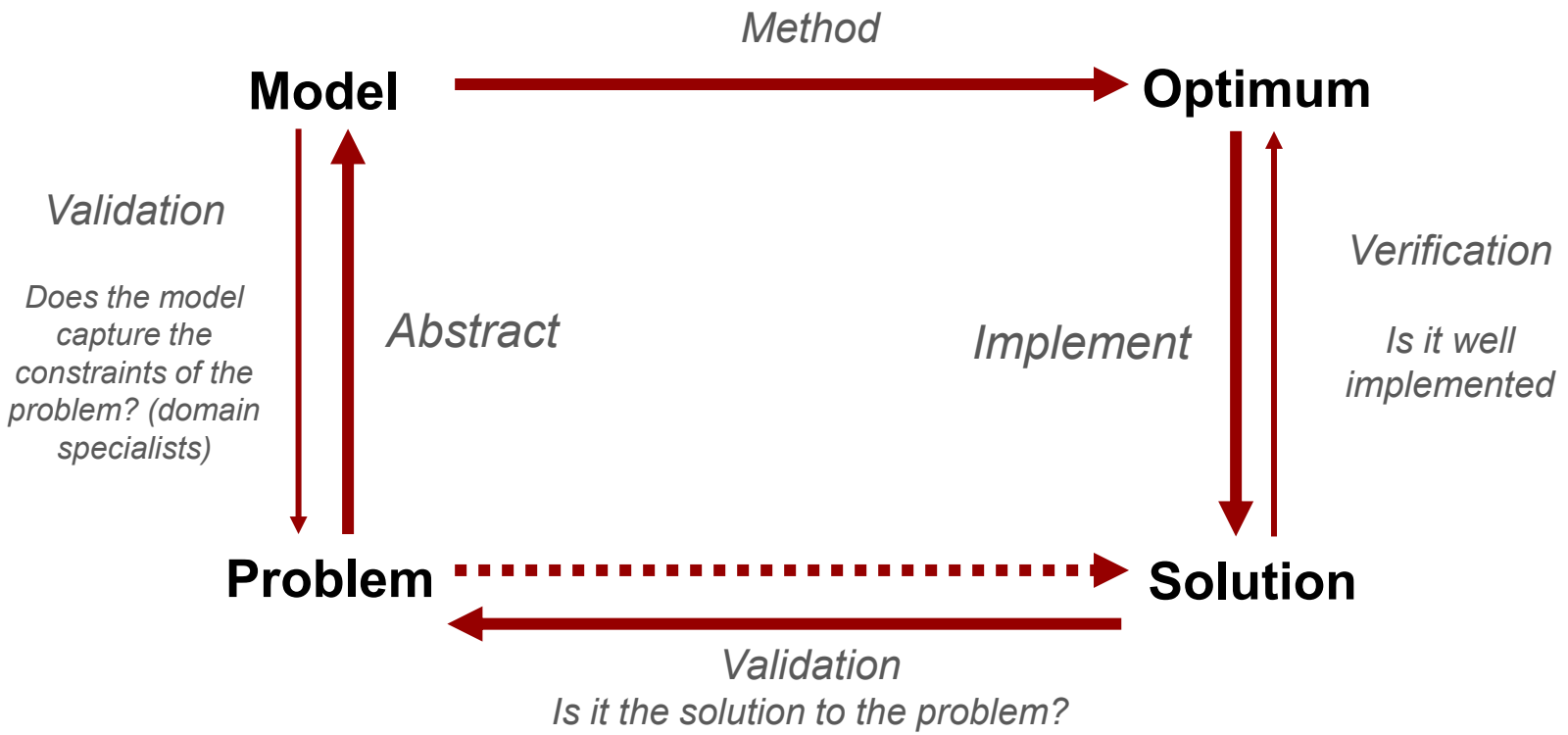
Optimization Workflow



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Break

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Why optimization?

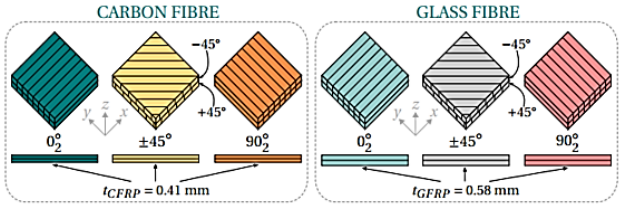
- Many problems require the identification of **the best (or the worst)** solution, decision or design.
- Usually misused. Example, “optimizing the design of a bridge” to refer to the *improvement* of the design, which is not necessary the best design, it is just better.

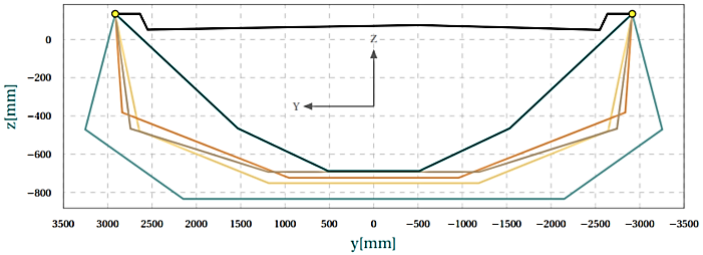
- **Applications**
  - As part of other mathematical tools, such as LS, MLE, ML, etc.
  - Direct applications: Optimal structural design, resource distribution, logistics, etc.

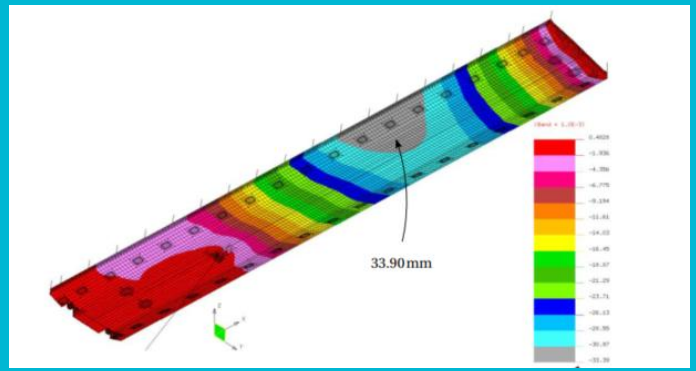
Let’s see some direct applications of optimization problems

Optimizing FRP (Fiber-Reinforced Polymer) bridges

- Goal: to identify the optimal **geometry and plies layout** with consideration of the tradeoff between cost and sustainability. The design must guarantee the structural safety conditions.







- MSc Thesis
- Ola Åsbø

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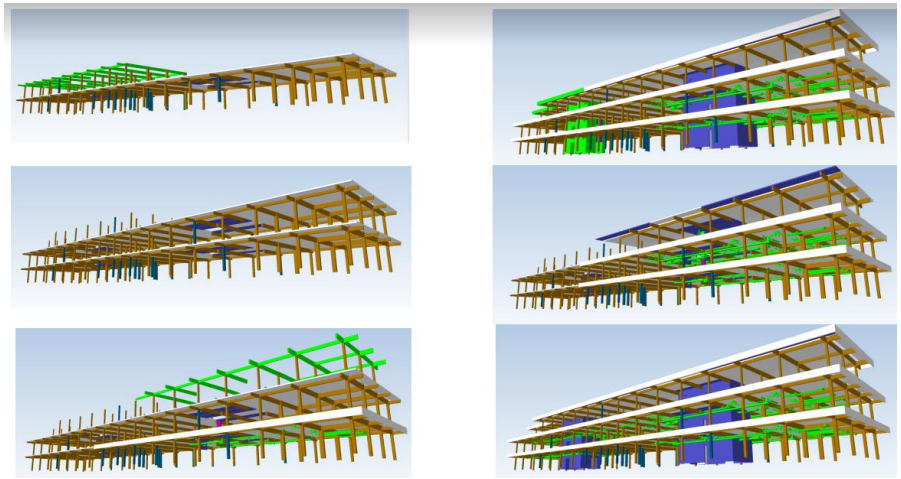
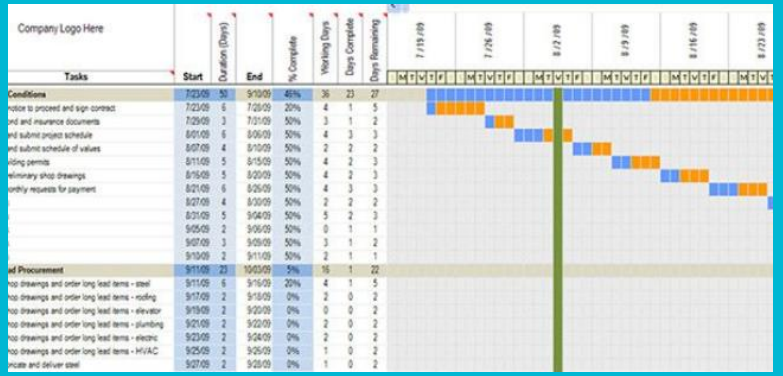
Optimal component-level construction schedule

- Goal: to determine the most efficient **process for building components** that have physical interdependencies (fulfilling constructive constraints), with consideration of the tradeoff between the cost and construction duration.



MSc Thesis - Xinzhi Jiang



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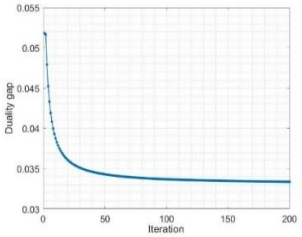
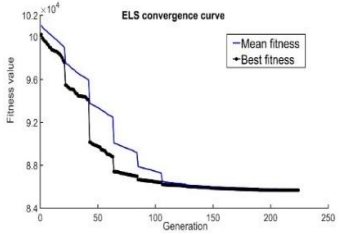
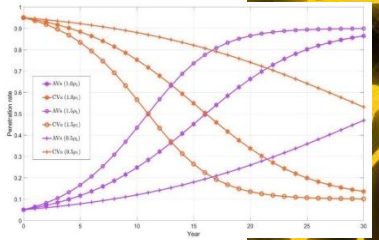
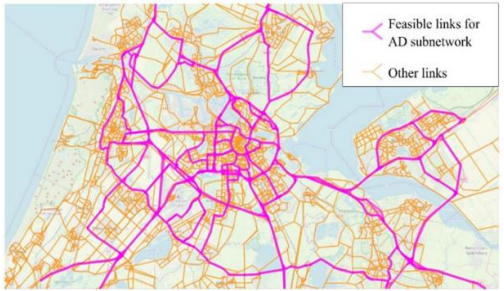
Designing road networks for automated vehicles

- Goal: To identify the best **location and time for deploying enhanced roads** for automated vehicles, minimizing deployment cost and maximizing efficiency and safety given the uncertain evolution path of automated driving technology and the travelers' mode and route choice behavior.





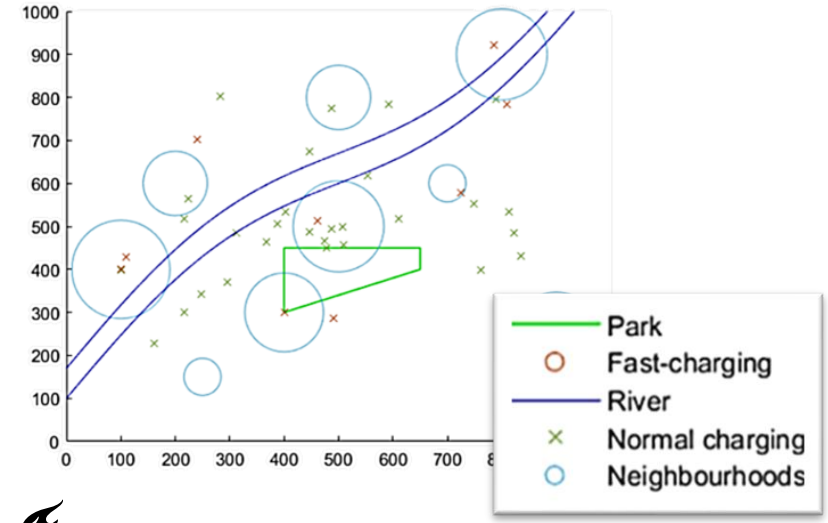




PhD thesis
Bahman Madadi

Optimization of electric vehicles charging station locations

- Goal: to identify the optimal distribution of charging stations that maximizes the utility accounting for the population density, existence of other charging stations, and other urban elements (e.g., rivers, parks…).



In groups of 2 – 3 discuss

- ❑ What is/are the objective function(s)?
- ❑ What is/are the decision/design variable(s)?
- ❑ What is/are the constraint(s)?

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CME4501 Engineering Systems Optimization - Final project

Tomas Raaphorst & Vitali van Elk

Optimization of electric vehicles charging station locations

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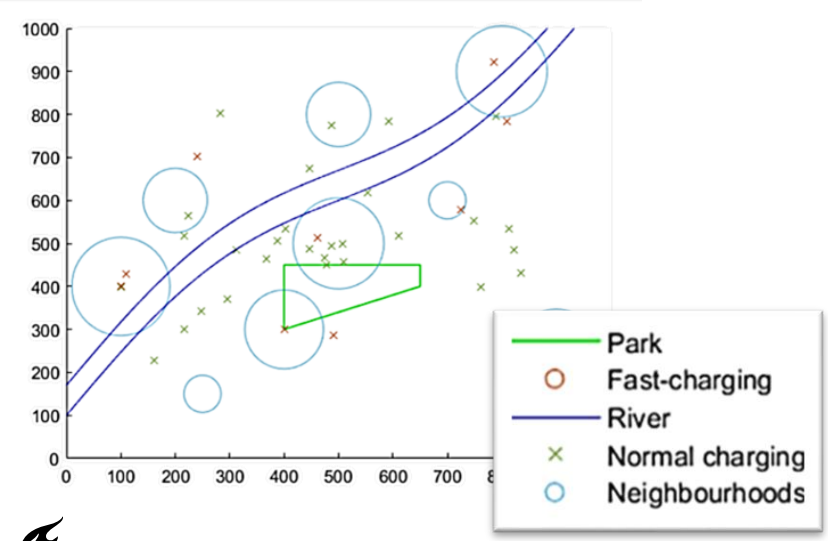

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What is the objective function?

✓ Maximize the utility (e.g., user accessibility, service coverage, etc.)

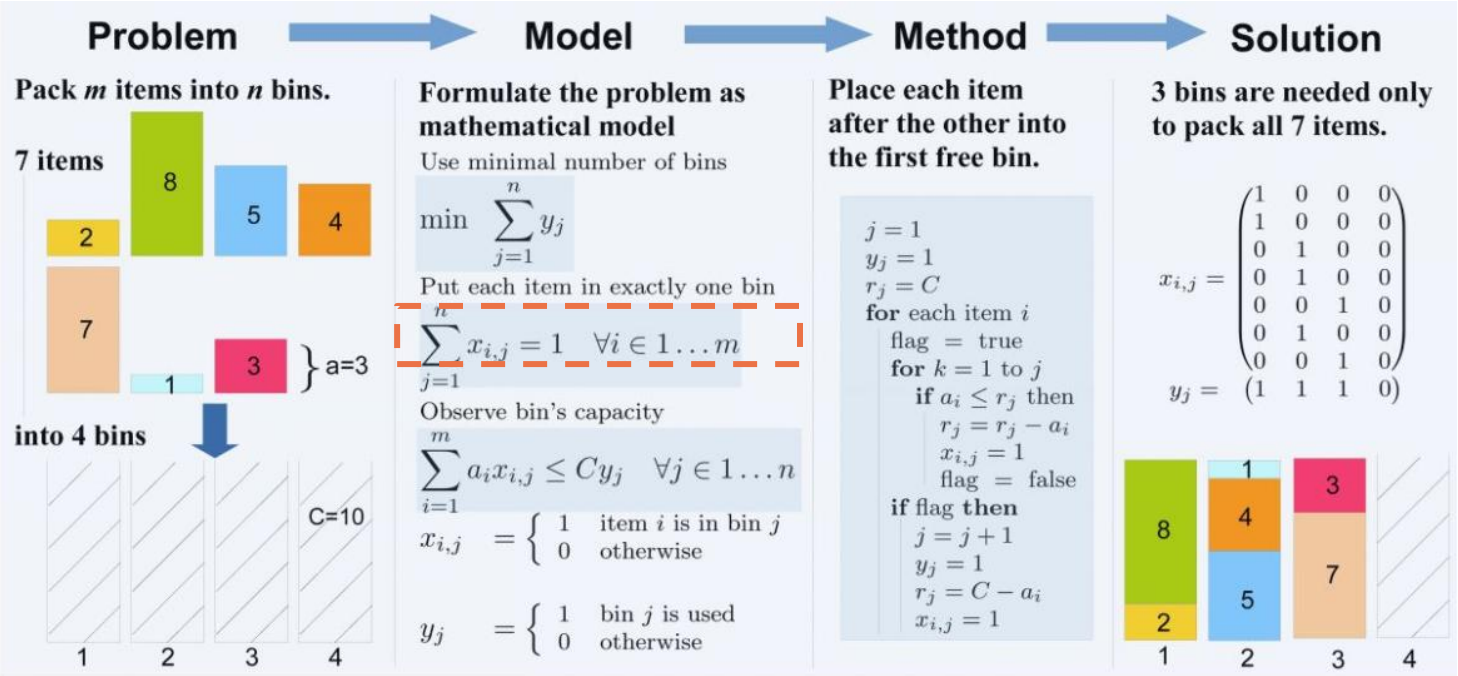
What is the decision/design variable?

✓ Distribution of charging stations (e.g., coordinates, 0/1 grid, number and average distance, etc.)

What are the constraints?

✓ Minimum charger-citizen ratio
✓ Minimum distance to other charging stations
✓ Maximum distance to other charging stations
✓ No chargers in a park, in a river, etc.
✓ …

What is included under the concept of optimization?



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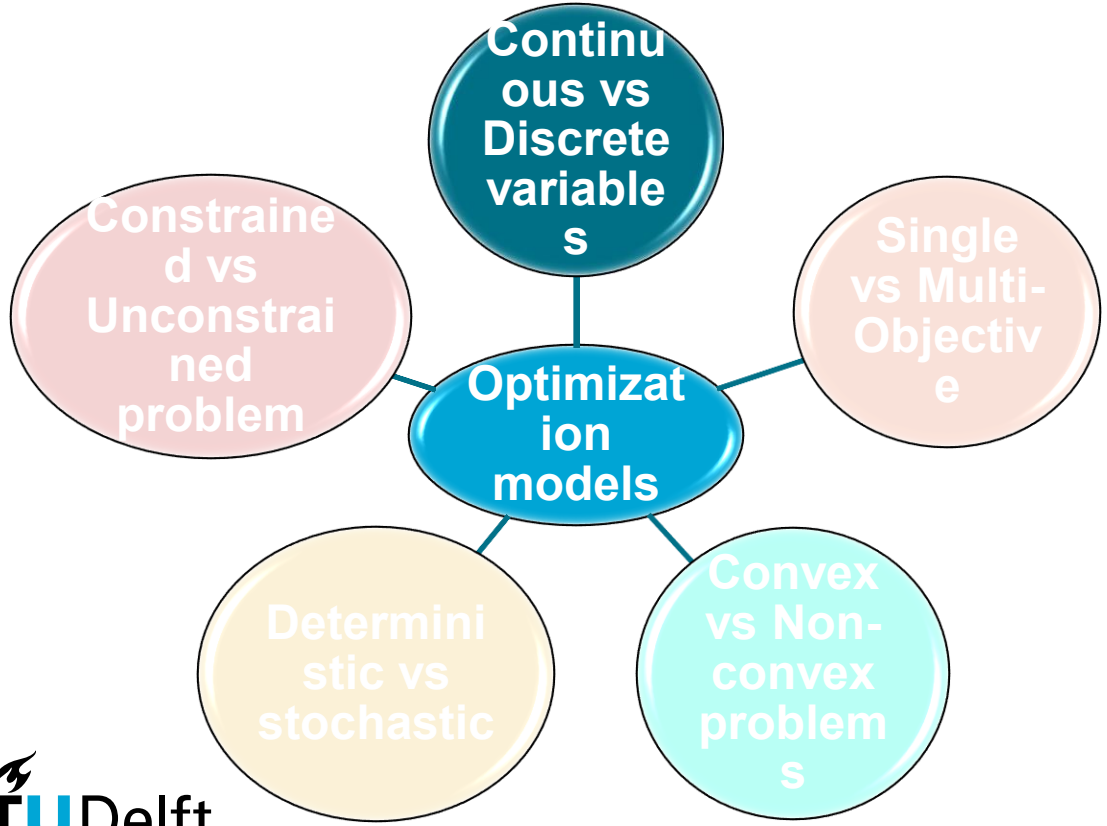
Optimization Models. Taxonomy



- **Continuous variables**:

  time, distances, physical properties, etc.

- **Discrete variables**:

  number of wind turbines, decisions such as doing something or not, type of materials, etc.

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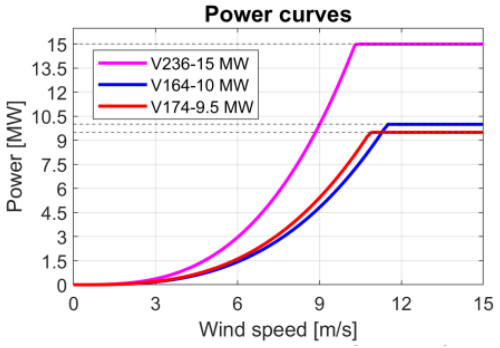
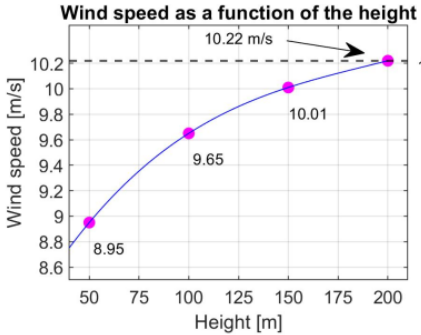
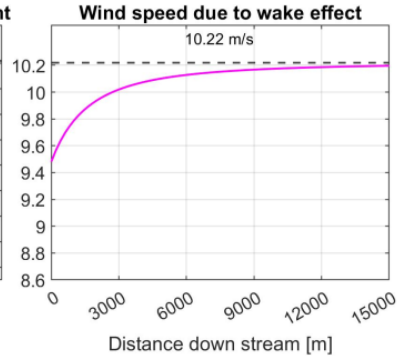
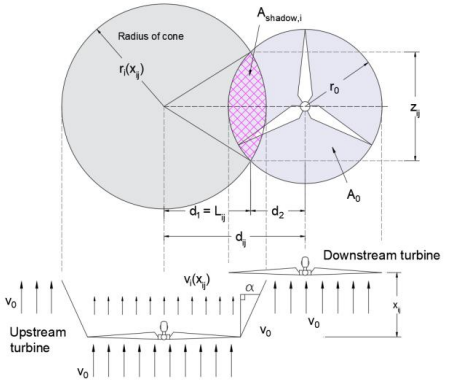
Optimizing the layout of the offshore wind farms in Norway

- Goal: Determine the optimal **layout of wind turbines** to produce the highest annual energy production with the minimum cost. Considering different types of turbines, their power curves, physical characteristics, wake effect, etc.









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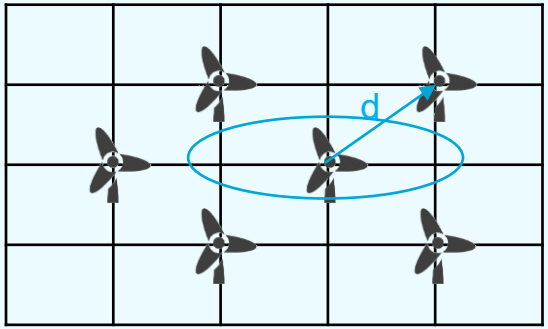
CME4501 Engineering Systems Optimization - Final project

J. Aulbers, O. Åsbø & I. Timori

Optimal Wind Turbine (WT) Farm



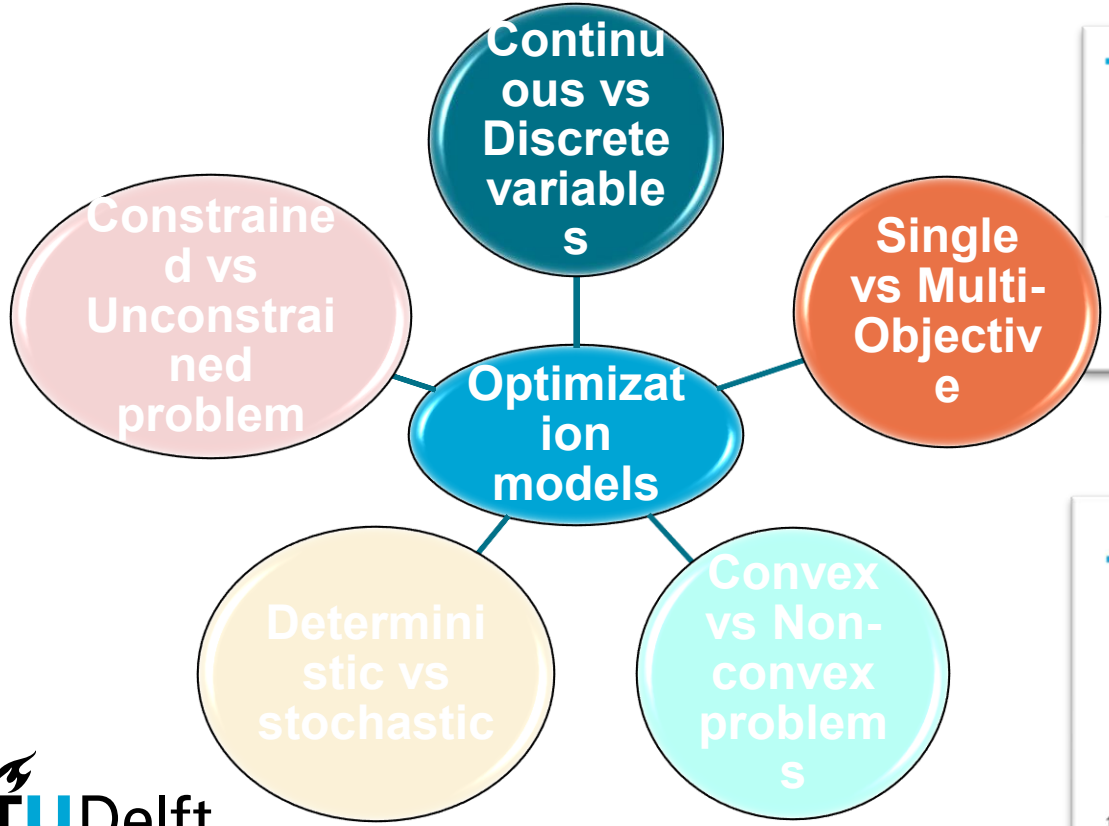
Discrete variables

Continuous variables

Decision variables:

- **n**: number of WTs (n is between 10 and 50).
- **d**: the closest distance between two WTs (d is between 15 and 100 m).

Optimization Models. Taxonomy



❖ **Single objective:**

- Goal: to identify the optimal **distribution of charging stations** that maximizes the utility accounting for the population density, existence of other charging stations, and other urban elements (e.g., rivers, parks…).

❖ **Multi-objective:**

- Goal: Determine the optimal **layout of wind turbines** to produce the highest annual energy production with the minimum cost. Considering different types of turbines, their power curves, physical characteristics, wake effect, etc.

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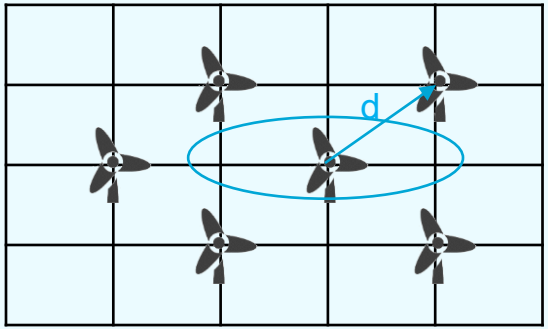
Optimal Wind Turbine (WT) Farm



Decision variables:

- **n**: number of WTs, [10,50]
- **d**: the closest distance between two WTs [15,100]

Objective function:

- Maximize the annual production:

  $\boldsymbol{Max_{n,d}\ n.P_{unit}(d)}$

  with $P_{unit}(d)$ being the energy production of 1 WT that depends on the distance between WTs

- Minimise the annual maintenance cost:

  $\boldsymbol{Min_{n,d}\ n.C_m}$

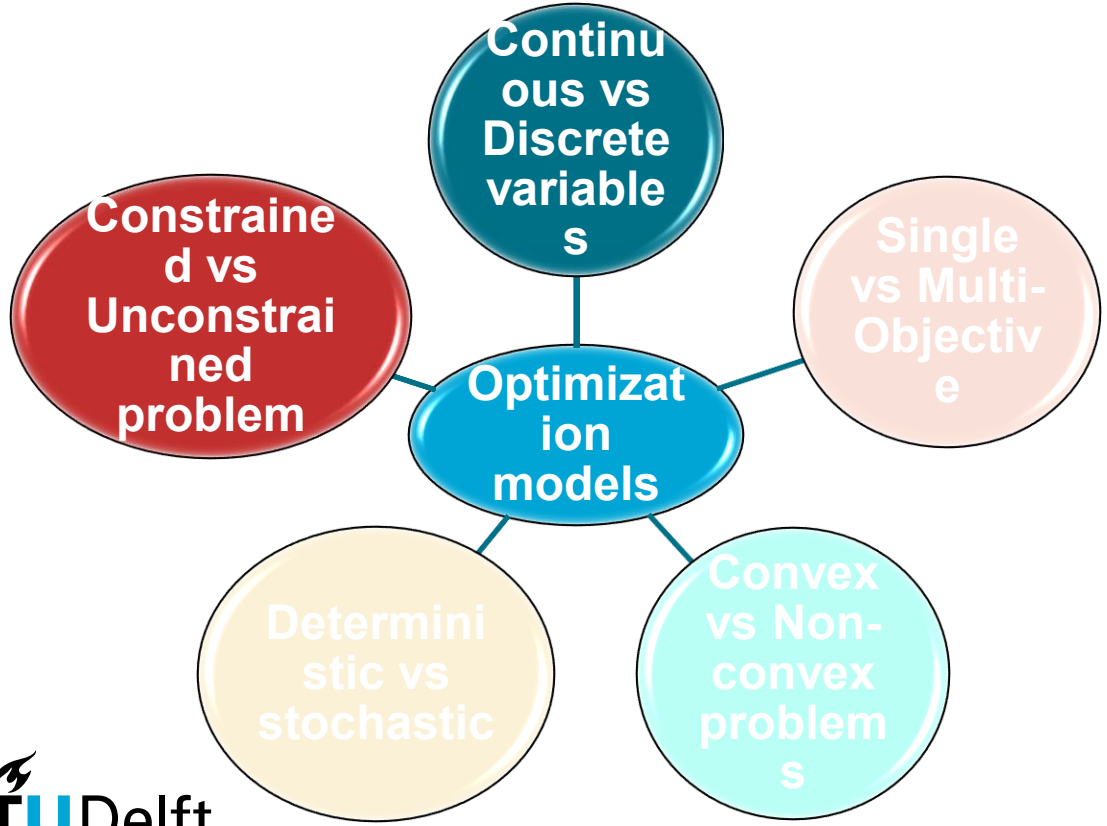
  with C_m being the annual maintenance cost of 1 WT

- Both objectives: $\boldsymbol{Min_{n,d}\ \{-n.P_{unit}(d);\ n.C_{unit}\}}$

Single objectives

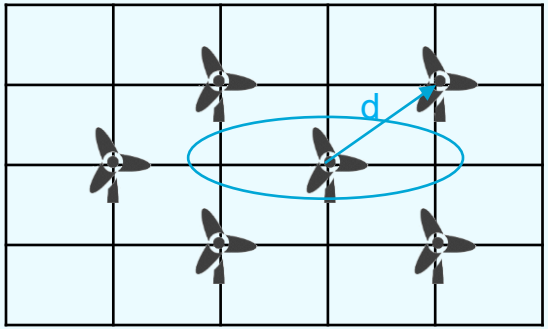
Multi-objective

Optimization Models. Taxonomy



- **Unconstrained problem:** the solution space is not bounded. All the configurations are possible candidates for being optimal.
- **Constrained problem:** the solution space is bounded.
  - Feasible region: Only a set of solutions are possible candidates.
  - Unfeasible region: There is not a possible solution fulfilling all the constraints.

Optimal Wind Turbine (WT) Farm



Decision variables:

- **n**: number of WTs, [10,50]
- **d**: the closest distance between two WTs [15,100]

Objective function:

- Maximize the annual production:
  $\boldsymbol{Max_{n,d}\ n.P_{unit}(d)}$

Constrained problem

If the construction (C) or maintenance costs (C_m) of a WT is larger than 5M or 0,07M EURO respectively, there is no possible solution. In such a case, we have an unfeasible problem.

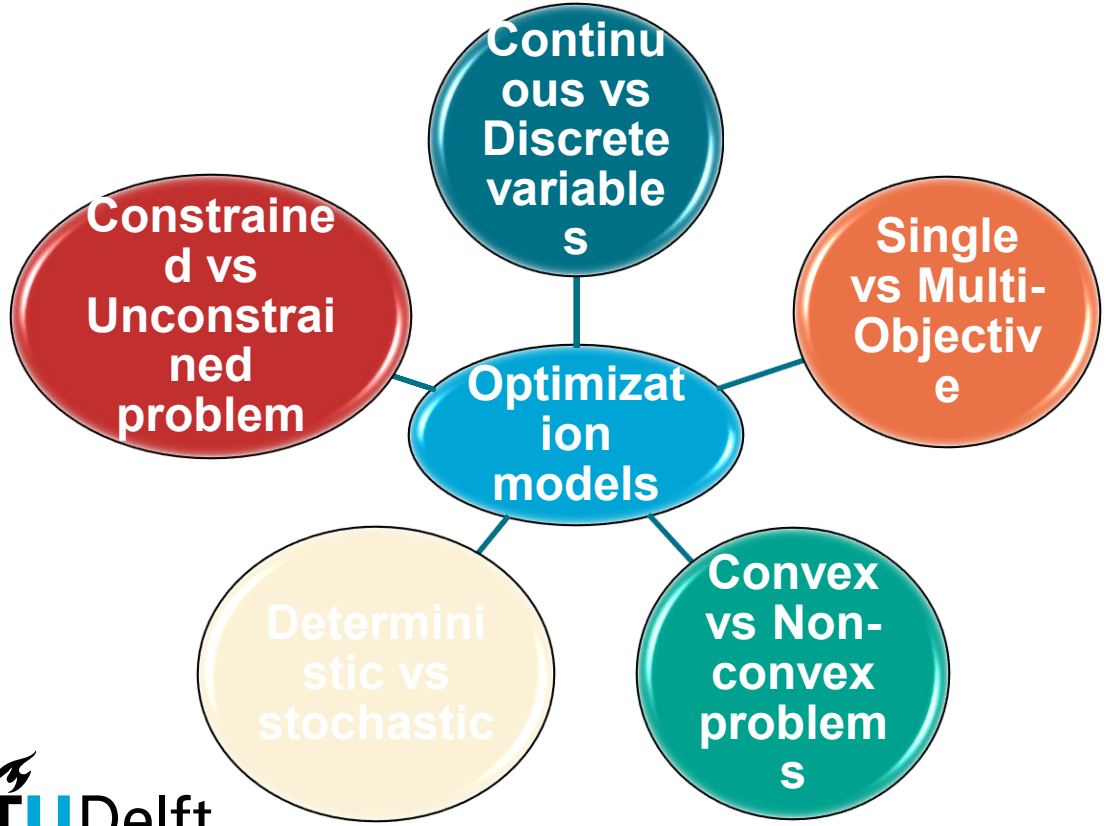
Constraints:

- Limited construction budget of 50M EURO:
  $\boldsymbol{n\,C \leq 50M}$
  with C being the construction cost of 1 WT

- Limited annual maintenance cost of 0,7M EURO:
  $\boldsymbol{n\,C_m \leq 0,7M}$
  with C_m being the annual maintenance cost of 1 WT

- …

Optimization Models. Taxonomy



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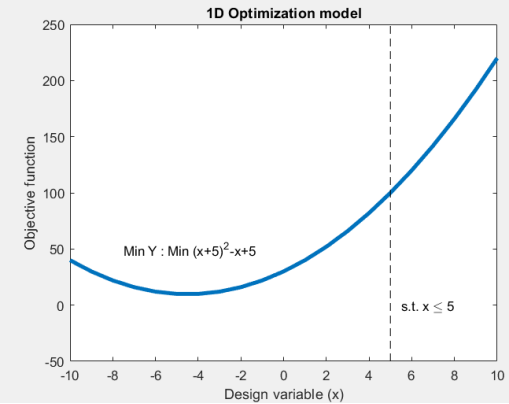
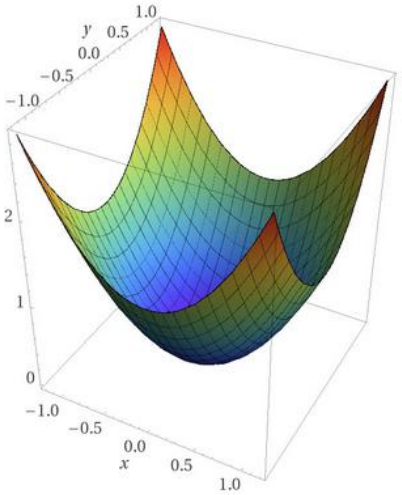
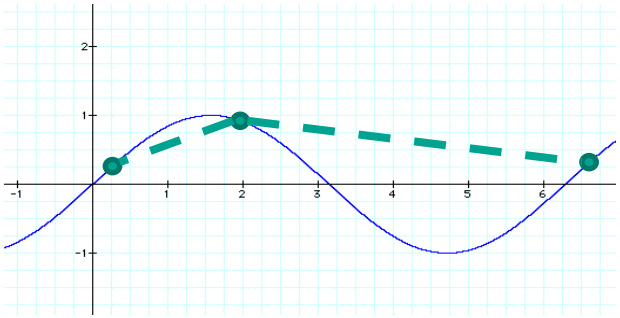
Convex problems

A convex optimization problem is a problem where all the constraints and the objective are convex functions

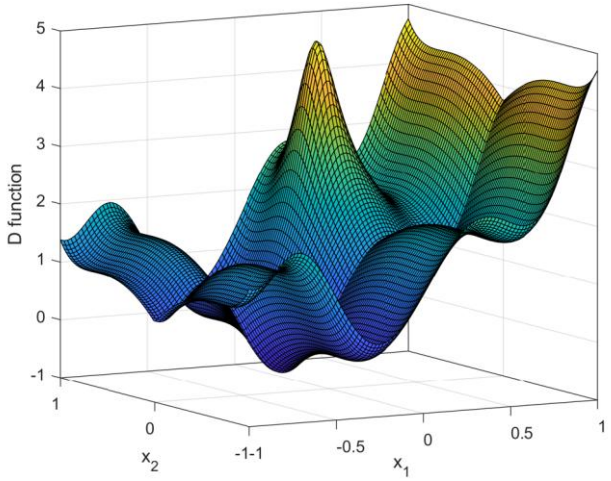


Convex function





Non-convex function



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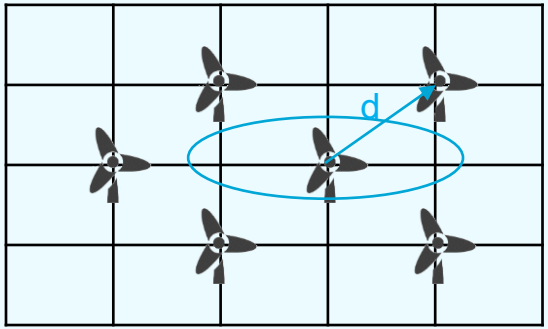
Optimal Wind Turbine (WT) Farm



Decision variables:

- **n**: number of WTs, [10,50]
- **d**: the closest distance between two WTs [15,100]

Objective function:

- Maximize the annual production: $\boldsymbol{Max_{n,d}\ n.P_{unit}(d)}$

Constraints:

- $\boldsymbol{n\,C \leq 50M}$
- $\boldsymbol{n\,C_m \leq 0,7M}$

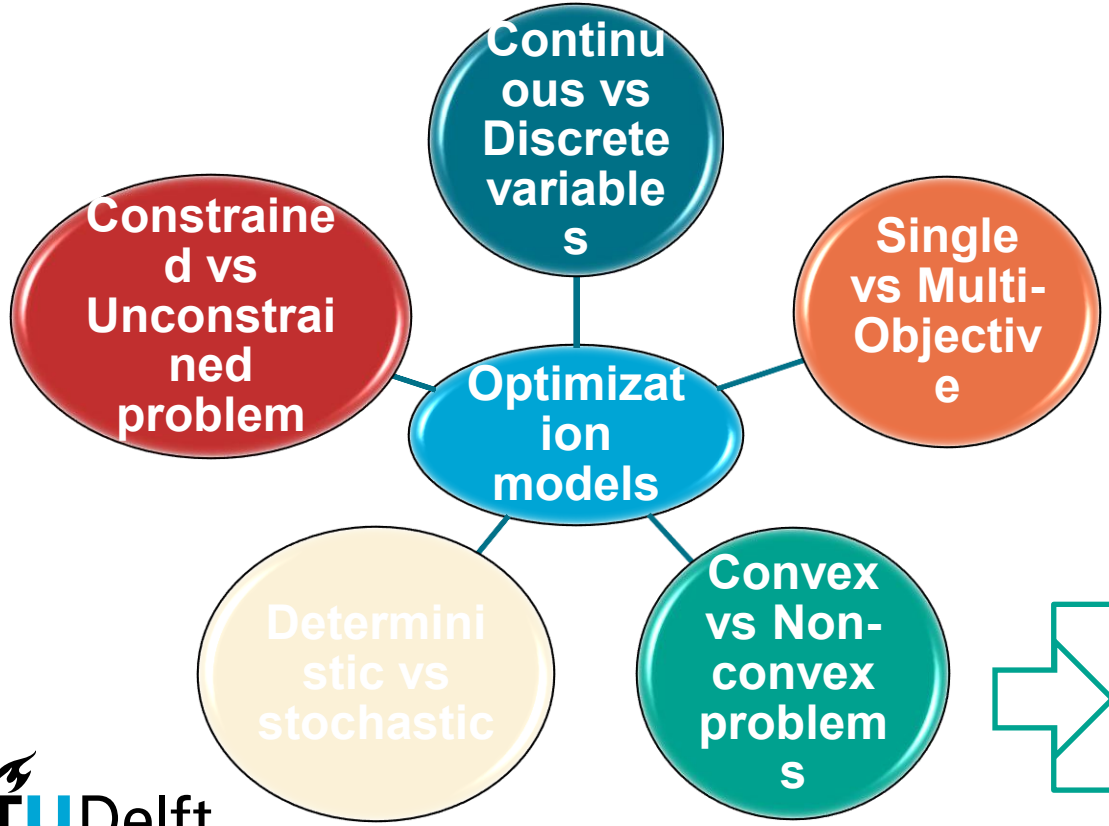
Linear = convex

Even if $\boldsymbol{P_{unit}(d)}$ were linear, the objective function would involve the product of two variables, thus, it is non-convex.

- If both, the objective function and all constraints are convex, the problem is convex.
- Linear and quadratic functions are convex.

✓ This optimization problem is NON-CONVEX because the objective function is non-convex

Optimization Models. Taxonomy



Common convex problem:

- Linear Programming (LP) ✓

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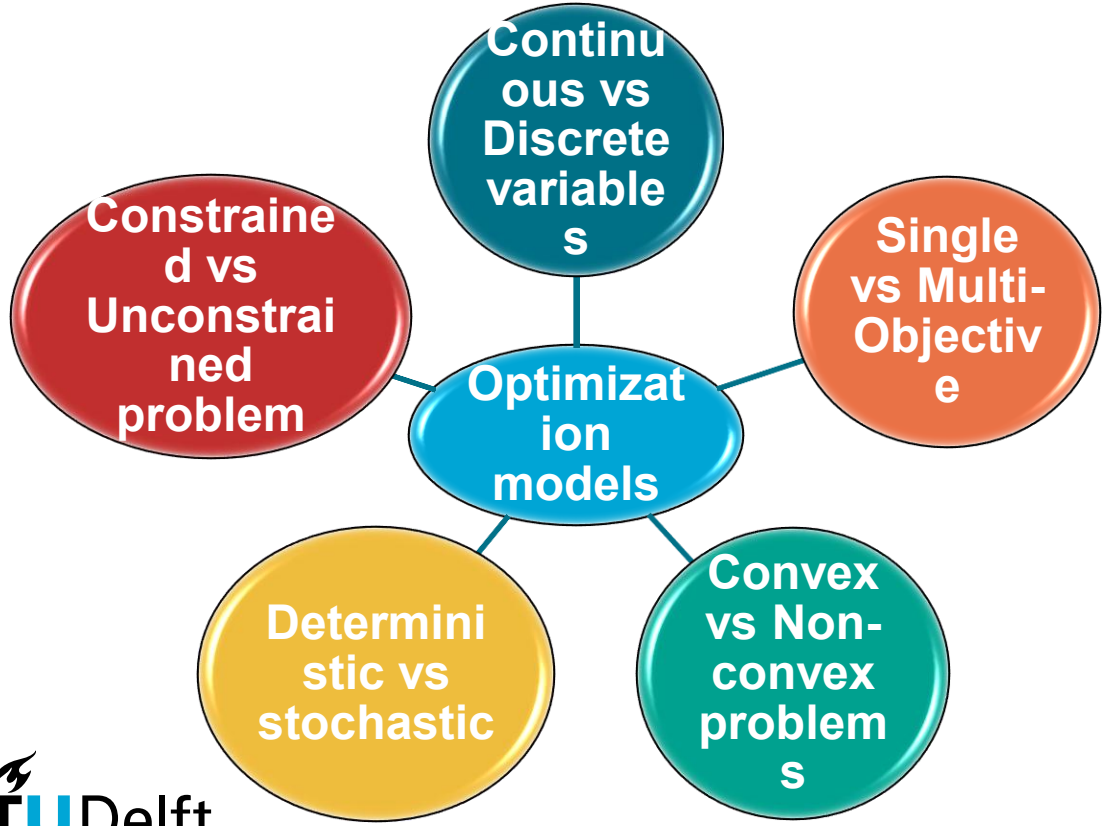
Optimization Models. Taxonomy



- **Deterministic optimization**: all the parameters of the optimization problem are deterministic. There is not variability in the problem definition.

- **Stochastic optimization**: The definition of optimization problem presents variability or uncertainty. The optimal solution of a possible scenario is not necessary the optimal solution of another possible scenario.

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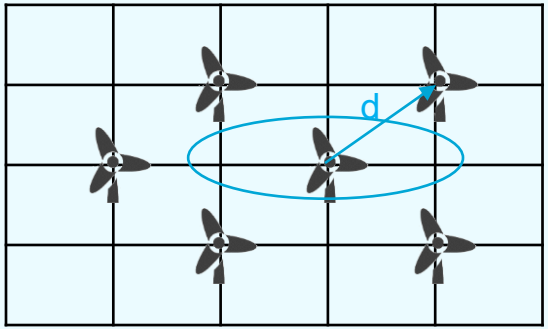
Optimal Wind Turbine (WT) Farm



Decision variables:

- **n**: number of WTs, [10,50]
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Objective function:

- Maximize the annual production: $\boldsymbol{Max_{n,d}\ n.P_{unit}(d)}$

Constraints:

- $\boldsymbol{n\,C \leq 50M}$
- $\boldsymbol{n\,C_m \leq 0,7M}$

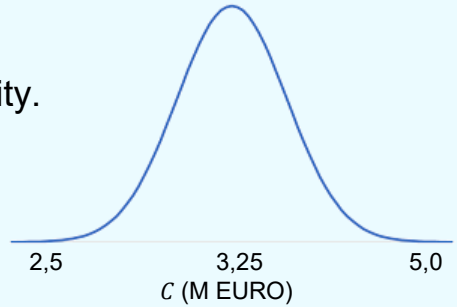
In case the construction (C) or/and maintenance costs (C_m) present variability. Then, the constraints can be expressed as

Deterministic

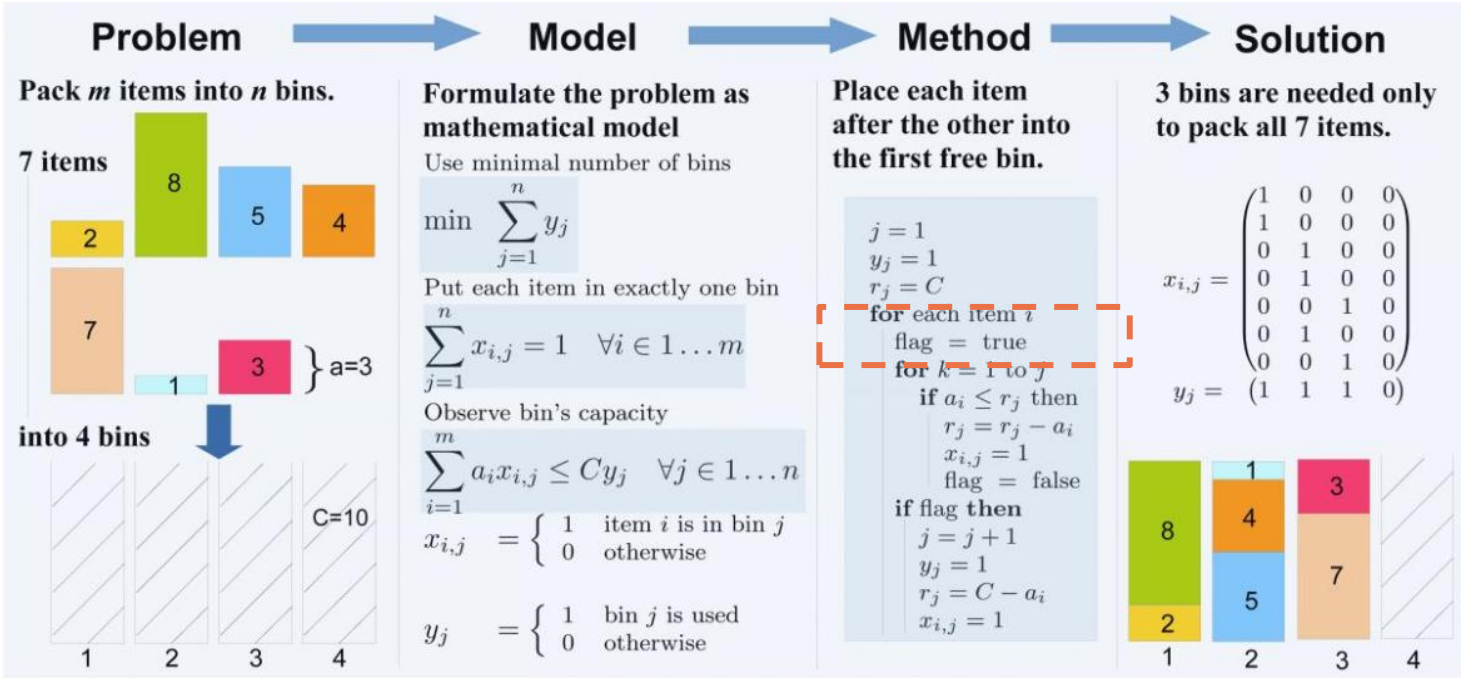
$$\boldsymbol{Prob(n\,C \leq 50M) \geq 0,90}$$

Stochastic

$$\boldsymbol{Prob(n\,C_m \leq 0,7M) \geq 0,95}$$



What is included under the concept of optimization?



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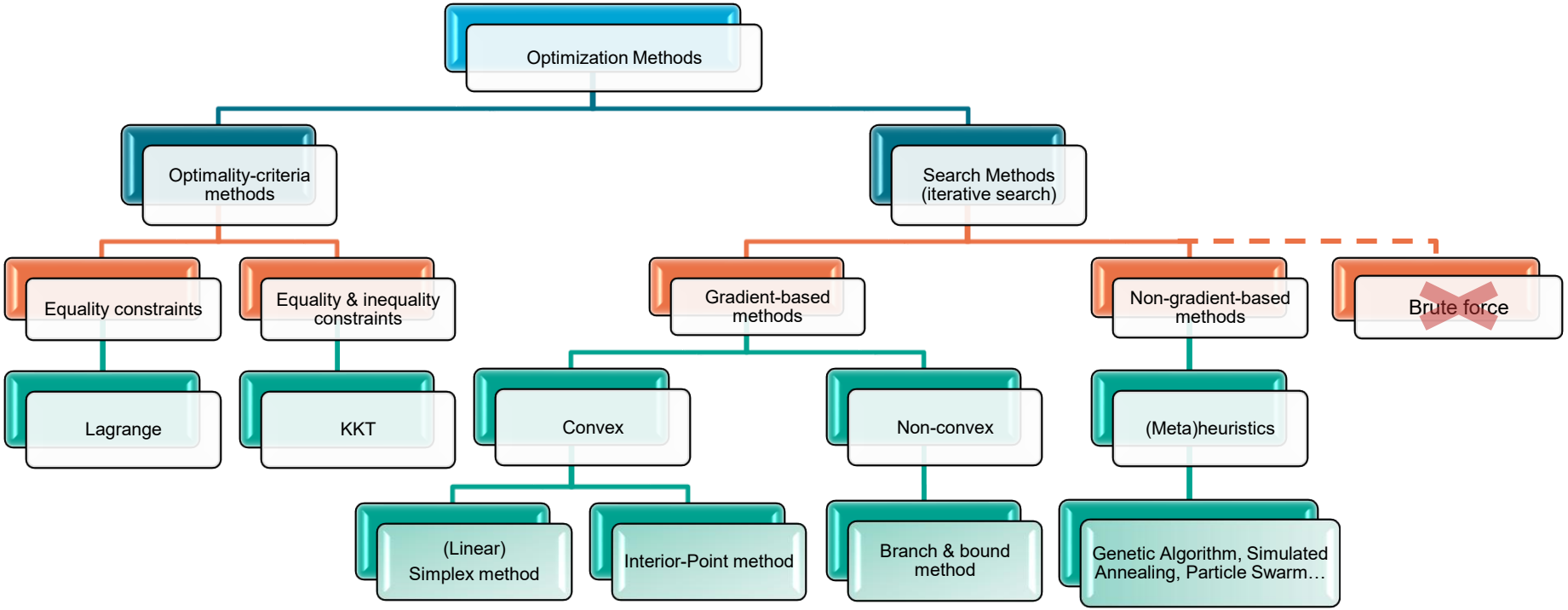
Optimization Methods. Taxonomy



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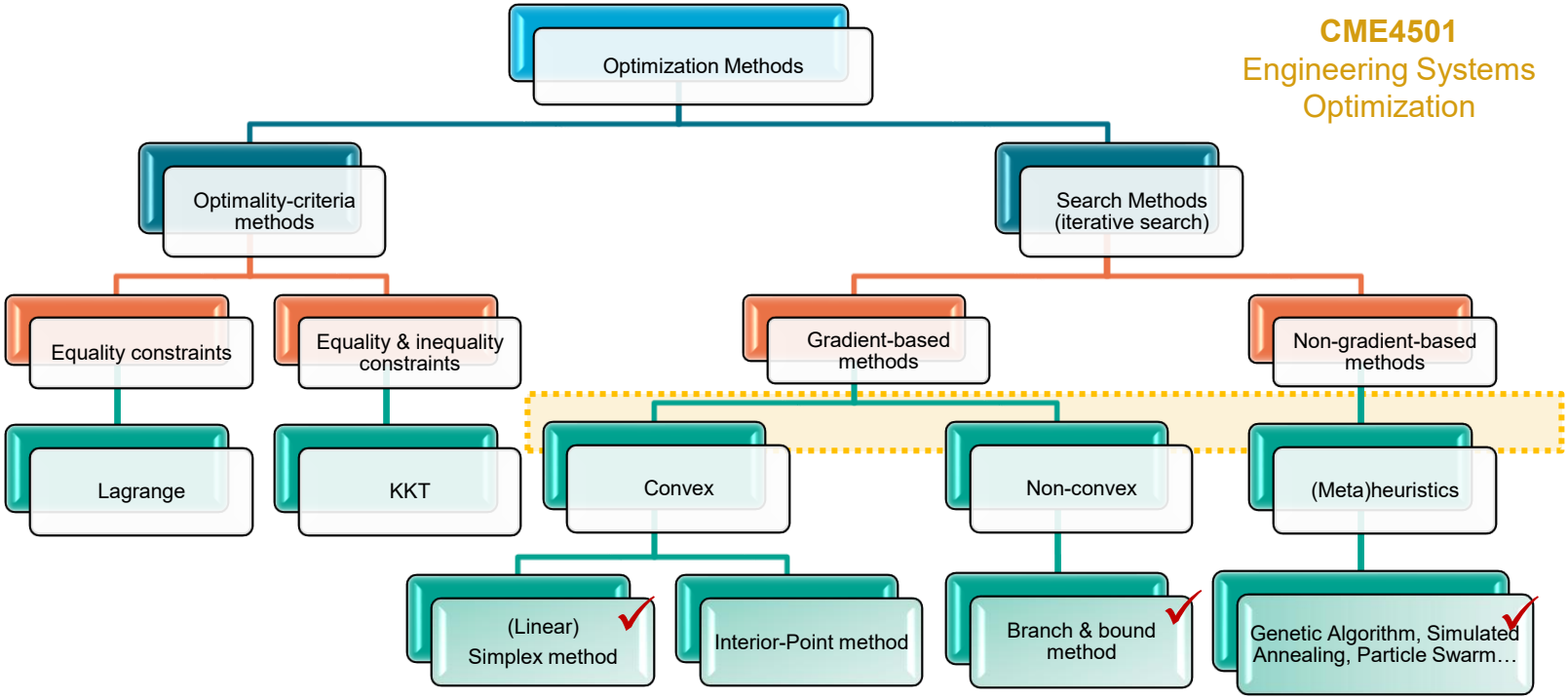
Level of complexity of the optimization model

Optimization Methods. Taxonomy



CME4501
Engineering Systems Optimization

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Questions!