A conceptual model of river ice breakup

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A conceptual model of ice breakup is formulated and used to analyze and compare data from four river gauge sites. Emphasis is on the development of generalized short-term forecasting methods, which to date have been site specific. The features to be forecast are the onset and flooding potential of breakup. These are related to the water surface width available for passage of the large sheets of ice that form by transverse cracking of the ice cover. Thus it is possible to study the effects of parameters such as ice cover dimensions and channel geometry. Owing to a lack of pertinent data, other parameters such as ice mechanical properties and flow characteristics are only considered indirectly. The mechanism of transverse cracking is examined in the light of recent field observations. Bending on planes parallel to the water surface, caused by stream curvature, could account for the observed crack spacing but more data are needed for positive conclusions. The present model does not apply in cases of "overmature" breakup, proximity of stage controls, and river planforms different from the single meandering channel type.

Key words: breakup, cracks, field data, forecasting, gauge records, ice, ice clearing, ice sheets, model, onset, river ice, rivers.

Un modèle conceptuel du bris du couvert de glace est formulé et appliqué à l'analyse et à la comparaison des observations de quatre stations de jaugeage. L'accent et mis sur le développement de méthodes générales de prévision à court terme, relations qui jusqu'à maintenant n'étaient mises au point que pour un site donné. Le début du bris et le potentiel d'inondation occasionné sont sujets à prévision. Ces variables sont associées à la largeur libre du cours d'eau disponible pour transiter de grandes plaques produites par le craquement transversal du couvert de glace. Il est alors possible d'étudier l'influence de paramètres tels que les dimensions du couvert de glace et la géométrie du cours d'eau. Toutefois l'absence de données pertinentes ne permet pas de prendre en compte directement d'autres paramètres tels que les propriétés mécaniques de la glace et les caractéristiques de l'écoulement. L'étude du mécanisme de craquement transversal est réalisée avec des données récemment recueillies en nature. La flexion, causée par la courbure du cours d'eau, dans le plan parallèle à la surface de l'eau peut expliquer l'espacement observé des craques mais des données additionnelles sont requises pour mener à des conclusions définitives. Le modèle formulé ne s'applique pas aux conditions d'un couvert de glace âgé, dont le bris n'est pas contrôlé par un phénomène mécanique, il ne s'applique pas à des zones sises dans le voisinage de points de contrôle du niveau de l'eau ni à d'autres cours d'eau que ceux de type chenal unique avec méandres.

Mots clés: bris du couvert de glace, craques, données observées en nature, prévision, données hydrométriques, glace, débâcle, plaques de glace, modèle, début du bris, glace de rivière, rivières.

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Introduction

A major consequence of ice cover formation in rivers is the jamming that occurs during the spring breakup of the cover and clearance of the ice from the river. Owing to their large thickness and hydraulic resistance, jams can cause unusually high water stages. This has repercussions in many operational and design problems, of which spring flooding is the most pressing. However, the present capability for engineering predictions related to breakup and jamming problems is very limited. Only crude estimates of jam stage are possible and only where it can be assumed that a floating equilibrium jam has formed nearby. Clearly, such information is hardly adequate for satisfactory consideration of practical questions such as short-term forecasting of the onset and severity of breakup, assessment of flooding frequency, and flood risk mapping.

A conceptual model of the breakup process is developed and discussed herein as a means of addressing short-term forecasting problems.

Background information

Shulyakovskii (1963) and Deslauriers (1968) have given excellent qualitative descriptions of the breakup process in rivers. The following brief description of breakup is based on these two references as well as on the writer's own experience. When an ice-covered river basin is subjected to mild weather, two processes generally begin: increased runoff due to rainfall or snowmelt or both, and heat input to the ice cover. The former process results in increased uplift and frictional forces applied on the ice cover and in increased water stage, which, in turn, reduces the support provided to the ice cover by the channel banks and provides increased channel width for movement of the cover. Heat input to the ice cover results in reduced dimensions and strength. It follows that, during the mild weather spell, the forces applied on the ice cover increase while the cover's ability to resist these forces decreases. If the mild weather lasts for a sufficient time, the ice cover begins to break up, which is often followed by the formation of large ice jams, major ice runs, and the eventual clearance of the ice from the reach of interest. This general description of the breakup process includes two extreme cases, i.e., the "premature" and "overmature" breakup (Deslauriers 1968). Premature breakup occurs under conditions of intense runoff with little, if any, deterioration of the ice cover. Clearly, this type of event has the greatest damage potential, other things being equal. On the other hand, conditions of slow or no runoff with intense ice deterioration lead to overmature breakup. This event is characterized by gradual ice disintegration and has minimal potential for damage.

Shulyakovskii (1963) proposed the following functional relationship for the onset of breakup:

$$[1] \quad \Sigma q = f(h_i, h_s, \Phi, V, H_0, H', \Sigma q_i)$$

in which Σq = total heat input per unit outer surface of the ice cover that is necessary to initiate breakup; h_i and h_s = ice thickness and snow depth, respectively, prior to the beginning of melting; Φ = (a set of) parameters describing stream morphology; V = average flow velocity; H_0 = initial water stage; H' = rise in water stage above H_0; and Σq_i = total heat input per unit inner surface of the cover. The rate of rise of the water level has also been mentioned as a pertinent parameter (e.g., Deslauriers 1968; Murakami 1972). Equation [1] is useful as a compact statement of the problem, but involves too many parameters to permit empirical assessment. For practical purposes, [1] is simplified by restricting attention to site-specific studies (Shulyakovskii 1963). This approach provides useful results but can only be applied to sites with good historical records. Since the publication of Shulyakovskii's book (1963), the state of the art has not advanced appreciably (see, for example, Murakami 1972 and Galbraith 1981). The forecasting indices used are empirical and often change from site to site. Clearly, substantial improvement can only be achieved by the development of a general conceptual model of the breakup process, which would lead to the quantitative relationship envisaged in [1].

A possible starting point is the following quotation (Shulyakovskii 1963):

> If the ice breakup develops during a rise in the water level, the stage "(H_B)" at which the ice push occurs is determined by the highest position of the ice cover during the winter, i.e., by the maximum winter stage "(H_F)."

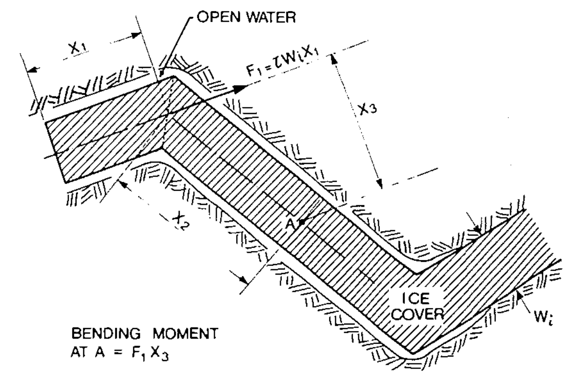

FIG. 1. Shulyakovskii's mechanism of transverse crack formation.

This suggestion will be discussed later and developed further. For the present, it is noted that use of H_B as an index of breakup initiation in conjunction with H_F not only appeals to intuition but has also been found satisfactory by the writer on several occasions. The following quotation is also pertinent (Gerard 1979):

> ...the only quantitative indication of the circumstances required to cause breakup of a reasonably competent ice cover is an increase in water level to near that which existed just after freeze-up the previous fall. Beyond this the moment of breakup is difficult to anticipate...

In a later work, Shulyakovskii (1972) presented a theoretical model of breakup initiation that sheds more light to the significance of H_B, as outlined next. The main force responsible for stresses in the ice cover is identified as the flow shear stress on the cover's underside.[1] The cover is assumed to be separated from the river banks by a distance dictated by the difference $H - H_F$ (H = prevailing water stage, say gauge height; H_F = maximum stable stage during the preceding freeze-up (see also later discussion)). The river is assumed to consist of linear segments intersecting at known angles (Fig. 1). Normal and bending stresses develop as illustrated in Fig. 1. Breakup initiation is defined as the instant when the strength of the cover is exceeded and transverse cracks form. By a simple structural analysis, confined to a plane parallel to the water surface, it is shown that breakup starts when

$$[2] \quad \sigma_i h_i = f_1(H_F, H_B)$$

in which σ_i = representative value of ice strength; H_B = stage at breakup initiation; and f_1 = a function whose mathematical form depends on channel geometry, flow velocity, and friction characteristics. If σ_i and h_i do not

[1] This should be increased by the streamwise component of the cover's own weight per unit area. Total driving force per unit area = τ.

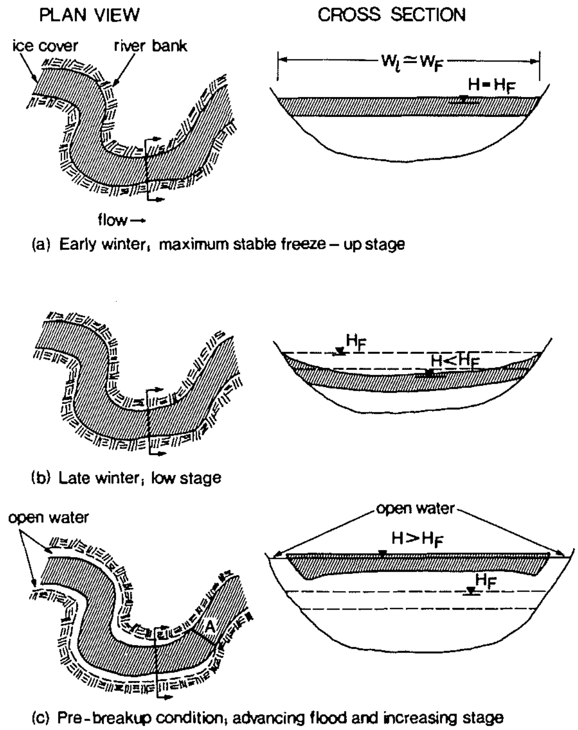

FIG. 2. Significance of H_F.

change appreciably from year to year, [2] reduces to

$$[3] \quad H_B = f_2(H_F)$$

which is in agreement with earlier findings of the same author (1963).

It is noted here that Shulyakovskii's (1972) concept ignores ice-breaking mechanisms that involve bending on vertical planes such as those studied by Michel and Abdelnour (1976) and Billfalk (1982).

A model of river ice breakup

A conceptual model of breakup is developed in this section based on existing understanding while introducing a few new conditions.

Significance of maximum stable freeze-up stage (H_F)

The significance of the maximum stable freeze-up stage, H_F, is illustrated in Fig. 2. Ice conditions in early winter, when the stage attains a maximum, are depicted in Fig. 2*a* (see also later discussion). The width of the ice cover can be approximated by W_F, the channel width at the stage H_F. To eliminate very brief maxima for which there is little time for freezing, H_F is defined as a daily average value. Later in the season, the stage drops and more ice may form, but the ice cover width does not change appreciably. So long as H remains less than H_F, the cover is supported by the channel banks. Under this boundary condition, the driving forces can be shown to produce very small stresses, not sufficient to break the cover. When warm weather and runoff start, a sufficiently high wave may travel downstream so that $H > H_F$ upstream of A (Fig. 2*c*) and $H < H_F$ downstream. Upstream of A, the cover may be considered a beam cantilevered at A. With the passage of time, A moves downstream and the stresses in the cover increase, leading to formation of transverse cracks.

The foregoing suggests that a stage in excess of H_F is a necessary condition for breakup. This is valid so long as the cover remains competent in thickness, width, and strength during the pre-breakup period. Though this occurs often in nature, there are instances of warm weather accompanied with insignificant runoff. The cover then deteriorates by thermal effects until it can either be broken by the available driving forces or slowly disintegrate in place. Deterioration may consist of full or partial loss of strength and reduction in ice thickness and width. This is the "overmature" breakup type as mentioned earlier. Another complication is the commonly observed formation of side cracks, usually preceding that of transverse ones. Side cracks (otherwise known as "hinge" cracks) are caused by uplift pressures which develop to accommodate the increased discharge. Side cracking is, to a degree, predictable and reduces the effective width of the cover. For simplicity, this effect will be temporarily ignored until certain basic relationships are demonstrated.

Description of the breakup process

Returning to the main discussion where it was seen how the first transverse cracks may be expected to form, it is noted that new cracks will appear as the flood wave advances. Eventually, a given river reach will be covered by large separate ice sheets. However, general breakup does not follow from this phase because the sheets may be too large to advance for any significant distance; they may be simply realigned into a "loose" but stable arrangement, as shown in Fig. 3 (see also later discussion). As the stage continues to increase, the water surface width increases until some of the sheets can "clear" the bends or other obstacles and move for a substantial distance. These sheets pick up speed and impact with stationary ones or with the channel boundaries, which causes fragmentation. Small jams begin to form, causing additional stage increases, new dislodgements, and so on, until the entire reach is cleared of ice. Based on this discussion, it is felt reasonable to define breakup initiation at a given site as the instant when a sustained movement of the cover takes place. This definition has the additional advantage of dealing with an easily observed event relative to transverse crack formation.

As breakup progresses, the destruction of the ice cover is accelerated by an increasing number of impacts

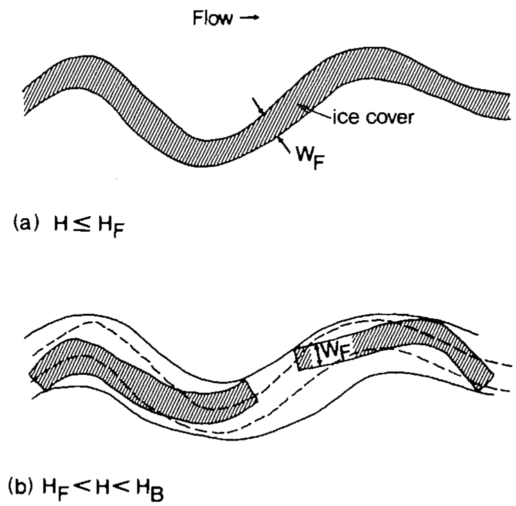

FIG. 3. "Loose" arrangement of large ice sheets.

and by thermal effects. The reach of interest will be cleared of ice when the sheet that holds the last ice jam is finally dislodged.

Dimensional analysis

The foregoing can be quantified as follows. Let l_i be a length representative of the longitudinal dimensions of the separate ice sheets illustrated in Fig. 3*b*. Breakup starts when the water surface width, W_B, is such that it "just" permits a sufficient number of ice sheets to clear the various obstructions. (Clearly, l_i will have a statistical distribution in a given reach rather than be a constant. The concept may be made more precise by stipulating that W_B is such that a fixed, though unknown, percentage of ice sheets are able to move. Then l_i will be the length corresponding to this percentage.) One could now write

$$[4] \quad W_B = f_3(W_i; l_i; L_1, \ldots, L_K; \theta_1, \ldots, \theta_n)$$

in which W_i = ice cover width; and L_K, θ_n = lengths and angles that define river plan geometry. By dimensional reasoning, [4] can be reduced to

$$[5] \quad W_B/W_i = f_4(l_i/W_i, \ldots L_K/W_i, \ldots \theta_n)$$

Figure 4 shows two examples for which [5] can be quantified. A curved sheet of average radius R and central angle θ will "clear" a straight reach when (Fig. 4*b*)

$$[6] \quad \frac{W_B}{W_i} = 1 + \left(\frac{R}{W_i} - \frac{1}{2}\right)\left(1 - \cos\frac{\theta}{2}\right)$$

It is noted that $\theta = l_i/R$ and [6] applies for $\theta < \pi$. The length l_i can be expressed as

$$[7] \quad l_i = f_5(\tau, \sigma_i, W_i, h_i, \ldots L_K, \ldots \theta_n)$$

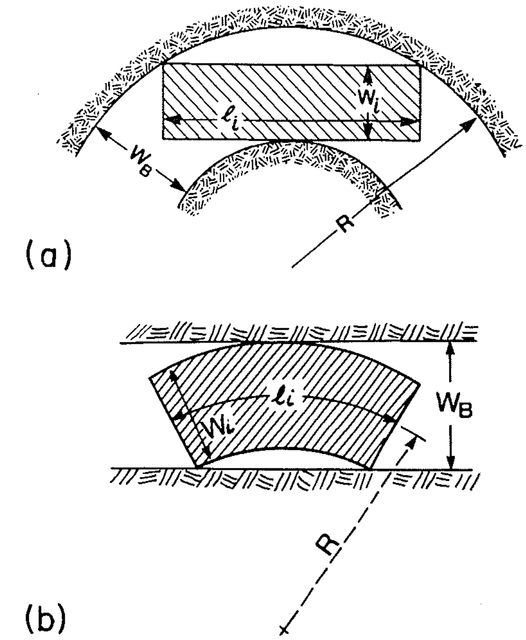

FIG. 4. Illustration of ice sheet movement threshold.

in which τ = driving force per unit area. Equation [7] implicitly assumes that the mechanism of transverse crack formation is as shown in Fig. 1. If another mechanism is assumed (e.g., approach of a steep water wave), some additional ice and water properties will be needed on the right-hand side of [7], as will be discussed later. However, this consideration does not alter the essence of what follows. Equation [7] can be nondimensionalized and substituted in [5] to obtain

$$[8] \quad W_B/W_i = f_6(h_i/W_i, \sigma_i/\tau, \ldots L_K/W_i, \ldots \theta_n)$$

Based on previous discussion, it can be assumed that $W_i \simeq W_F$, provided the cover has not been subjected to significant side cracking and melting. Because W_F varies little from year to year (freeze-up flows), the parameters L_K/W_i could be considered river constants as a first approximation. Moreover, in most natural streams, W varies as a power of Y (= average depth) so that W_B/W_i can be replaced by the more practical parameter Y_B/Y_F. With these assumptions, [8] reduces to

$$[9] \quad Y_B/Y_F = f_7(h_i/W_F, \sigma_i/\tau; \text{dimensionless river constants})$$

From the physical understanding described so far, it is reasonable to expect that the function f_7 increases with both h_i/W_F and σ_i/τ. The dimensionless river constants serve to account for the channel plan geometry. For example, the L_K's may be used to identify such dimensions as meander length and amplitude, radii of curva-

TABLE 1. Summary of gauge site characteristics

Site description	Type of data	Latitude	Slope (m/km)	Long-term mean conditions*: Width (m)	Depth (m)	Discharge (m^3/s)
Thames R. at Thamesville	Gauge records, 1960–1979; observations, 1980–1982	42°32′42″N	0.23	37	2.0	49
Nashwaak R. at Durham Bridge	Gauge records, 1965–1981	46°07′33″N	0.73	58	1.2	36
Peace R. at Peace River	Observations, 1974–1976, 1979	56°14′41″N	0.35	470	3.5	1800
Smoky R. at Watino	Observations, 1976–1979	55°42′56″N	0.52	225	1.9	370

*Width and depth values are for open-water conditions at the long-term mean discharge. For Peace and Smoky Rivers, data were obtained from Kellerhals *et al.* (1972).

ture of bends, lengths and widths of islands, etc., while the θ_n's identify typical bend angles.

The second major problem associated with breakup is how to forecast its severity, which can be partly quantified by H_m, the maximum breakup stage. It was mentioned earlier that a reach will be cleared of ice when the ice sheet that is least amenable to dislodgement is eventually lifted to a level at which it can advance. Letting Y_c be the average flow depth at this level, one could write, as before,

$$[10] \quad Y_c/Y_F \leq f_8(h_i/W_F, \sigma_i/\tau; \text{dimensionless river constants})$$

The inequality symbol has been used because the last ice sheet to move will very likely be subjected to reductions in competence and dimensions during the breakup period. The depth Y_c can be expressed in terms of the corresponding discharge intensity q_c via a resistance formula and substituted in [10] to obtain

$$[11] \quad \frac{(q_c^2/gS)^{1/3}}{Y_F} + \frac{1.59}{f_c^{1/3}}\left(\frac{s_i h_i}{Y_F}\right) \leq \frac{1.59}{f_c^{1/3}} f_8\left(\frac{h_i}{W_F}, \ldots\right)$$

in which g = acceleration of gravity, S = channel slope, s_i = specific gravity of ice, and f_c = composite friction factor of the flow under the ice sheet. Equation [11] implies that there exists an "ice clearing" discharge, of which the upper limit depends on H_F, h_i, σ_i, τ, and channel geomorphology. In all probability, the last ice sheet to move will be holding back an ice jam whose potential stage can be estimated in terms of q_c and channel hydraulics (Beltaos 1983; Pariset *et al.* 1966). This places an upper limit on H_m, independent of discharge but dictated by H_F, h_i, σ_i, τ, as well as channel geomorphic characteristics.

To test the foregoing analysis, data from several sources have been used, as described next.

Description of data

Apart from relatively few data obtained by direct observation, the major data sources have been Water Survey of Canada records at hydrometric gauge sites (see Table 1). These include stage records, discharge measurement notes, and local observers' reports on ice conditions. Supplementary information consists of meteorological data and channel hydraulics obtained by hydrometric surveys in the gauged reaches. From these data, several parameters have been extracted, as outlined below (see also Beltaos (1984) for a more complete description of data extraction procedures).

Maximum stable freeze-up stage H_F.—A typical, though not universal, configuration of the daily average stage hydrograph at the start of the ice season is sketched in Fig. 5. While the effective stage (= stage that would have occurred had the flow been unaffected by ice) decreases continuously, the actual stage is seen to first rise, reach a peak, and then decrease again. The rise is caused by the upstream advance of the ice cover, formed by the jamming of slush pancakes at some point downstream of the gauge. Once the ice cover edge arrives at the gauge site, the stage begins to drop owing to decreasing discharge and thermal smoothing of the underside of the cover.

"Winter" peaks.—Occasionally, a brief thaw may occur during the winter, resulting in a peak on the stage hydrograph. If this peak does not initiate a breakup, it can be considered a lower limit for H_B at that time.

Stage at initiation of breakup, H_B.—When a thaw does lead to breakup, the stage hydrograph shows irregularities that cannot be explained by discharge vari-

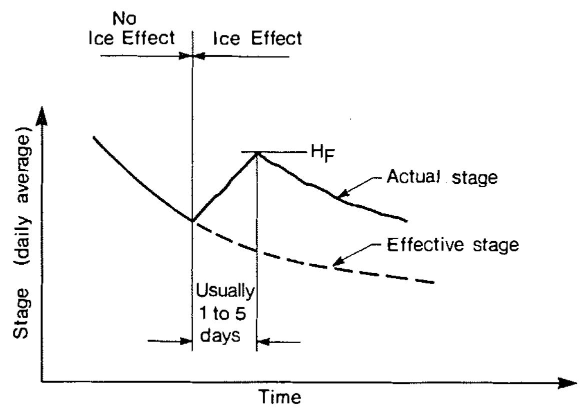

FIG. 5. Schematic illustration of daily stage variation with time during beginning of freeze-up.

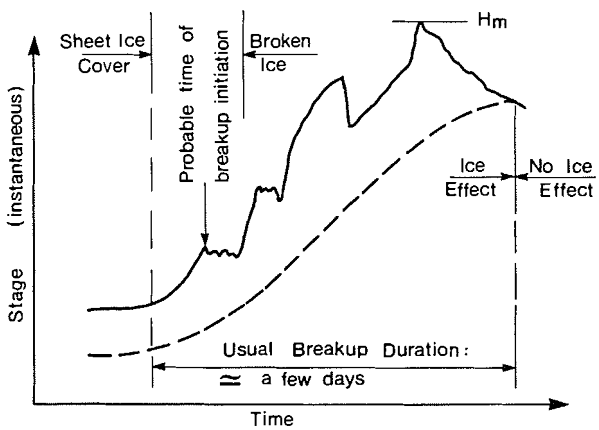

FIG. 6. Schematic illustration of instantaneous stage variation with time during breakup.

ations, as illustrated in Fig. 6. A probable value for H_B may be fixed at the first significant spike or slowdown in the rate of stage rise. (It may be recalled here that breakup initiation has been defined as the instant when a sustained movement of the ice cover begins. When a stationary ice sheet is set in motion, the stage would tend to drop as a result of reduced resistance to flow. This effect may be masked, however, by simultaneous rapid increases in discharge.) However, this definition is not always objective or meaningful. Only a probable range of H_B can then be determined by considering (a) the latest time for which it can be confidently assumed that there still was continuous ice cover and (b) the earliest time for which broken ice effects became evident. Simultaneous consideration of local observers' reports and familiarity with local conditions greatly increase the accuracy of H_B determinations.

Maximum breakup stage, H_m.—The determination of H_m is straightforward (Fig. 6).

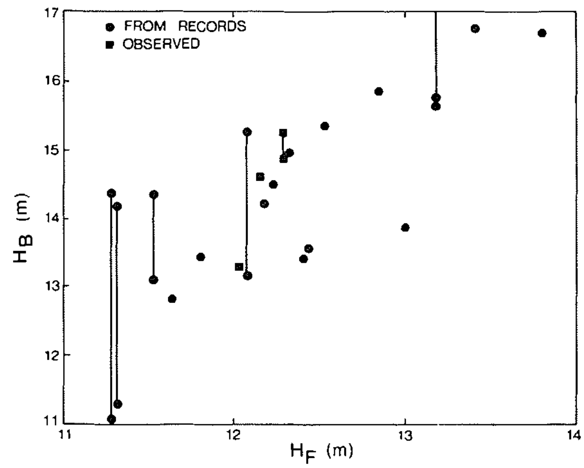

FIG. 7. Effect of H_F on H_B, Thames R. at Thamesville.

Discharge, Q.—Daily average discharges are estimated by Water Survey of Canada, based on interpolations between measurements and on such evidence as upstream and tributary flows, runoff, and weather conditions. Such estimates may involve large errors during breakup, except where discharge measurements have been performed, as is the case for most of the years of record at the Thames River gauge. For the other gauges in Table 1, discharge values used herein must be viewed as, at best, crude.

Ice thickness, h_i.—Ice thickness can be approximately determined from discharge measurement notes, subject to certain limitations (Beltaos 1984). For years without measurements, h_i can be estimated from site-specific correlations between measured values and time since the date of H_F. Such values of h_i are designated herein as "estimated" and involve errors of up to 30%.

Model testing—basic variables

In this section, the dimensionless relationships derived earlier are tested in a preliminary manner, i.e., by ignoring the effects of σ_i, τ, and side cracking, which will be discussed in the next section. Figure 7 shows a trend for H_B to increase with H_F. The effect of h_i is considered in Fig. 8 where a trend for $H_B - H_F$ to increase with h_i is exhibited. Also shown in Fig. 8 are data for the Smoky and Peace Rivers, plotted as ranges due to limited variability. The latter do not fit the Thames River relationship. Figure 9 shows the same data sets plotted in the dimensionless form suggested by [9]. Despite the scatter, the anticipated trend is confirmed. More importantly, the Smoky and Peace Rivers data are now much more consistent with the Thames River data than they were in Fig. 8. The scatter in Fig. 9 can be partly attributed to variations of σ_i/τ, which, however, are unknown because neither σ_i nor τ values

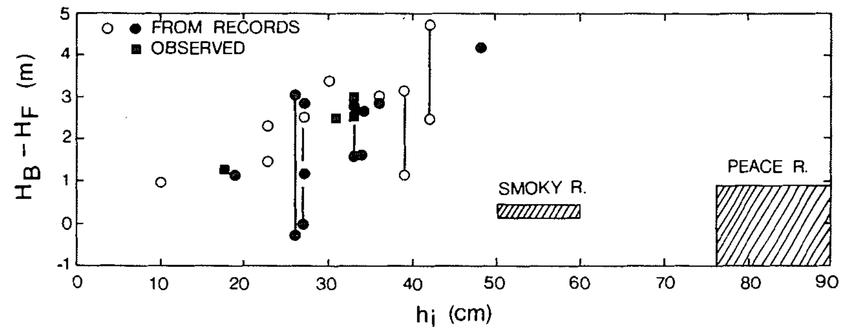

FIG. 8. Effect of ice thickness on $H_B - H_F$; data points are for Thames R. at Thamesville, open circles denote estimated h_i's.

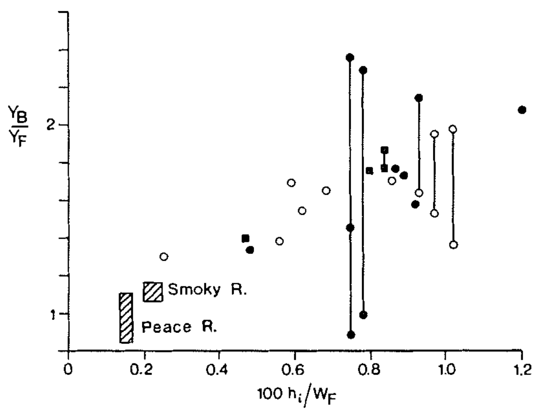

FIG. 9. Test of [9], Y_B/Y_F versus h_i/W_F; legend same as for Fig. 8.

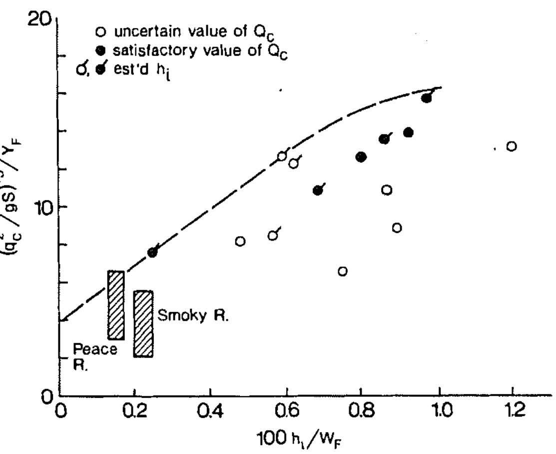

FIG. 10. Dimensionless "ice-clearing" discharge versus dimensionless ice thickness.

are available. It is also noted that the present findings can explain the empirical results of Murakami (1972) (see Beltaos 1982). A similar analysis for two rivers in the United States resulted in a set of data points consistent with those shown in Fig. 9 (D. Calkins, personal communication); the range of $100h_i/W_F$ for this set was from 0.4 to 2.0.

To test the predicted existence of the "ice clearing" discharge, the maximum discharge, Q_m, attained during the breakup period has been used. To determine the corresponding discharge intensity, q_m, the water surface width downstream of the last ice jam just prior to its release is needed. This is unknown for the present data but setting $q_m \simeq Q_m/W_B$ is considered a fair approximation. For the Thames, Peace, and Smoky Rivers, h_i is small relative to Y_c, hence the second term on the left-hand side of [11] has been neglected. With these assumptions, data for these three rivers are plotted in Fig. 10, in the form suggested by [11]. The data points define an upper envelope that exhibits the anticipated dependency on h_i/W_F. From earlier discussion, it is reasonable to expect that the function f_8 in [10] takes a value of 1.0 when h_i/W_F vanishes. Inspection of [11] and Fig. 10 suggests further that the intercept ($\simeq 4.0$) at $h_i/W_F = 0$ should be equal to $1.59/f_c^{1/3}$. This gives $f_c \simeq 0.063$, which is a plausible friction factor value for a channel covered with sheet ice at the time of breakup (corresponding Manning coefficient $\simeq 0.032$). Additional support for the ice-clearing discharge concept is provided by Figs. 11 and 12, where H_m is seen to be influenced by both H_F and h_i in the expected manner. The scatter is thought to be due to (a) whether or not the ice-clearing discharge is realized in any one breakup event and (b) as yet unknown effects of thermal and mechanical deterioration. At a given site, a graph such as Fig. 10 could be utilized as follows: Let q_m = peak discharge intensity, forecast for a runoff event expected to cause breakup; and q_c = value obtained from the upper envelope in Fig. 10. If $q_m \leq q_c$, then H_m should not exceed the potential jam stage for $q = q_m$. However, if $q_m > q_c$, H_m should not exceed the jam stage for $q = q_c$.

Model testing—other effects

A preliminary comparison of data with the present

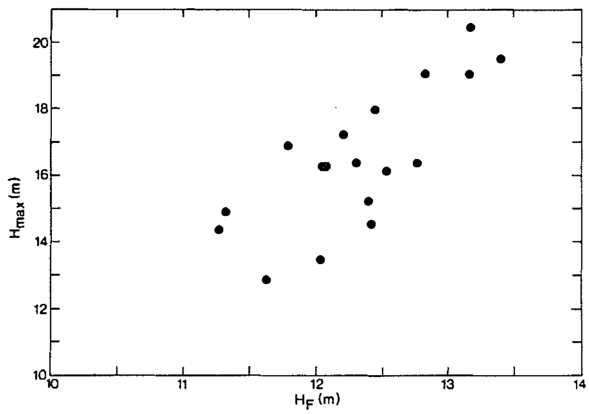

FIG. 11. Maximum breakup stage versus H_F, Thames R. at Thamesville.

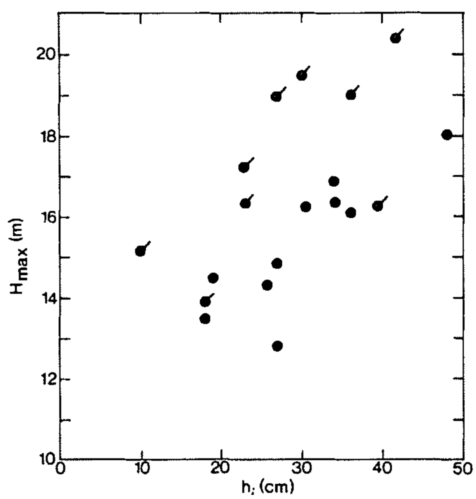

FIG. 12. Maximum breakup stage versus h_i, Thames R. at Thamesville.

model has produced encouraging results. However, there remain several questions that need addressing. For example, what is the effect of side cracks? Is the pre-breakup pattern postulated in Fig. 3 realistic? What are the effects of σ_i and τ? Is the model a general one or just one of several different breakup processes? These questions are considered in this section.

Effects of side cracks

A floating cover attached to the channel banks and subjected to uplift pressures may be considered a beam on an elastic foundation (Billfalk 1981). Using the appropriate structural theory (Hetenyi 1946) it is possible to predict the uplift pressure head (ΔH) required to cause side cracking and the locations of the side cracks. Billfalk (1981) performed this calculation for infinitely wide channels and showed good agreement with mea-

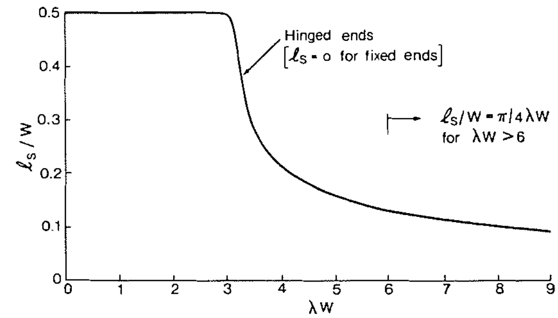

FIG. 13. Location of side cracks.

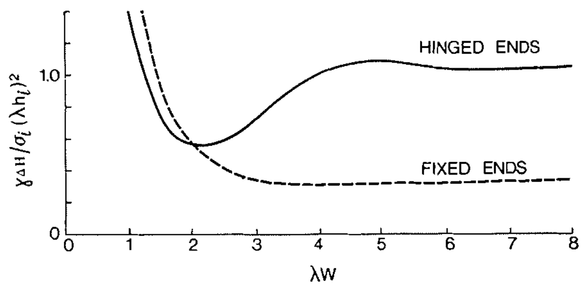

FIG. 14. Dimensionless uplift pressure head required to cause side cracking.

surements. The type of support assumed for the ice edges has a large effect on l_s (= distance of side cracks from respective edges). For fixed ends, $l_s = 0$. For hinged ends, it was found that

[12] $\lambda l_s = \pi/4$ (infinitely wide channel)

in which λ is defined as

[13] $\lambda \equiv \sqrt[4]{\gamma/4E_iI}$

with γ = unit weight of water, E_i = elastic modulus of ice, and I = moment of inertia per unit cover width = $h_i^3/12$. Billfalk's analysis has been extended to the finite-width case (Beltaos and Wong, unpublished data) and the results are shown in Figs. 13 and 14. For hinged ends, the two side cracks merge into a single central crack for $\lambda W \leq 3$, while [12] applies for $\lambda W \geq 6$. Figure 13 affords a means of applying a correction to W_F in order to determine W_i. To match observed with predicted l_s's for the Thames River, a value of E_i = 1.4 GPa was found appropriate. This is considerably less than 6.8 GPa, representing good-quality, freshwater ice covers (Gold 1971). The difference could be due to thermal deterioration and creep effects; it is much less pronounced when comparing predicted (with 6.8 GPa) and observed l_s's because l_s varies as the fourth root of E_i.

To illustrate the effect of the side crack correction, W_B/W_F and W_B/W_i (W_i = corrected width) are re-

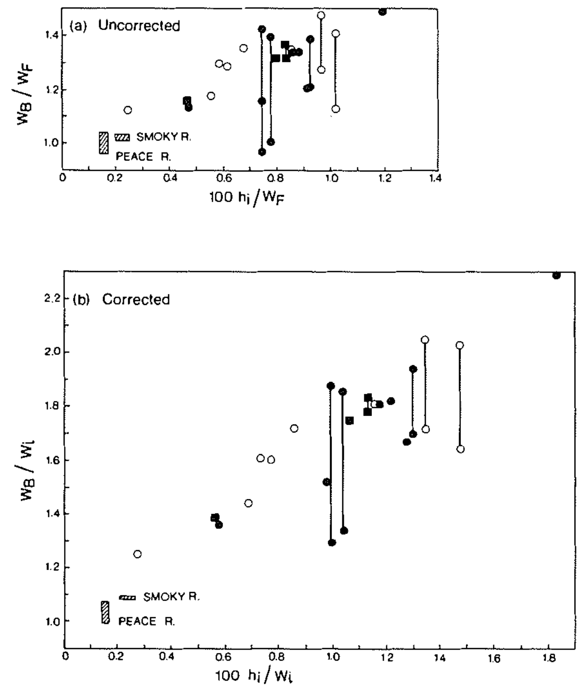

FIG. 15. Effect of side-crack correction, Thames, Peace, and Smoky Rivers; legend same as for Fig. 8.

spectively plotted versus h_i/W_F and h_i/W_i in Fig. 15 (see eq. [8]). A reduction in scatter seems to be effected by this correction. Figure 16, comprising the Nashwaak River data (see Table 1), provides a more striking illustration. Whereas the uncorrected width plot shows no trend (Fig. 16*a*), the width correction effects a clear increasing trend (Fig. 16*b*), similar to that found in Fig. 15. At the same time, it may be observed that the Nashwaak River data exhibit more scatter and a smaller rate of increase with h_i/W_i than the Thames River data. These differences will be discussed later.

Observed transverse crack patterns

Because the possible significance of transverse cracks was only recently understood, only one documentation of their spacing and location can be furnished herein, as shown in Fig. 17. The centre of the reach shown in this figure is located some 25 km downstream of the Thamesville gauge site. Crack locations are approximate because they were viewed from a height of 400 m and drawn on a 1:50 000 map. Nevertheless, Fig. 17 shows a fairly consistent crack spacing, reminiscent of the conditions postulated in Fig. 3*b*.

For the reach shown in Fig. 17, it is estimated that W_i = 55 m and h_i = 0.35 m. Therefore, $100h_i/W_i \simeq 0.64$ which, from Fig. 15, gives $W_B/W_i \simeq 1.46$. The photographs of Fig. 17 indicate that the water surface to ice cover width ratio was less than 1.46, which agrees with the fact that breakup had not yet been initiated.

A frequency analysis[2] indicated that the median ice sheet length in Fig. 17 was 300 m while the 16- to 84-percentile values were 225–415 m. Corresponding values of l_i/W_i are 5.5, and 4.1–7.5. These can be shown to be comparable to what is implied by earlier findings, as follows. Using [6] as a rough guide and putting W_B/W_i = 1.46, one can solve for l_i/W_i, provided R/W_i is known. For the sheets of Fig. 17, it was found that $R/W_i \simeq 6.2$ (median) and 3.2–15.6 (16- to 84-percentiles). With these values, [6] gives $l_i/W_i \simeq 5.0$ (median) and 3.8–7.7 (16- to 84-percentiles).

Mechanism of transverse crack formation

Shulyakovskii's (1972) postulated mechanism has been illustrated in Fig. 1 where it was assumed that the river comprises linear segments of uniform width. A slight improvement that avoids this "linearization" resulted in (see Fig. 18)

$$[14] \quad M = 2\tau W_i a_M$$

in which M = bending moment at C; and a_M = area of segment ABC. A transverse crack will form at C when $6M/h_iW_i^2$ becomes equal to σ_i (= flexural ice strength). By virtue of [14], this condition leads to

$$[15] \quad a_M = \sigma_i h_i W_i / 12\tau$$

Equations [14] and [15] involve the following assumptions: (a) ice cover curvature effects on the stress distribution at C are negligible, which is a good approximation for mild curvature (Flugge 1962); (b) the elementary force $\tau W_i ds$ (Fig. 18) acts on the centre line of the cover, which, too, is a good approximation for mild curvature; (c) contributions to the stresses at C by normal forces are negligible, which is estimated to apply in most instances; and (d) moments caused by forces that may be transmitted between adjacent sheets (e.g., at A in Fig. 18) are ignored (see also later discussion).

Equation [15] shows that a_M and thence l_i must vary along a given reach owing to changing planform geometry and ever-present variations in σ_i, h_i, W_i, and τ. For the ice sheets shown in Fig. 17, the following average values have been estimated: W_i = 55 m, h_i = 0.35 m, τ = 5.0 Pa, and a_M = 6400 m^2. Substituting these in [15] gives σ_i = 20 kPa, which is very low relative to 600 kPa, a common flexural strength value for good-quality ice as measured by the well-known cantilever or simply supported beam tests (Frankenstein 1961; Korzhavin 1971; Butyagin 1972). This large discrepancy is moderated by the following considerations:

(a) The value of σ_i has been found to decrease with specimen size. Using empirical results (Butyagin

[2]After transferring the observed crack locations to twofold enlargements of 1:10 000 vertical air photos.

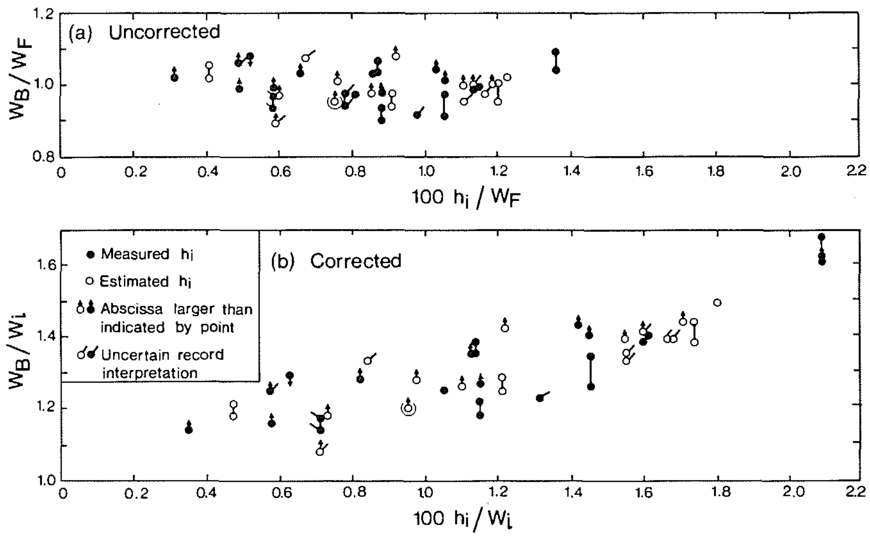

FIG. 16. Effect of side-crack correction, Nashwaak R.

1972), a reduction factor of at least three was estimated for the flexural strength of the entire ice cover cross section, relative to that obtained from beam tests. This would bring σ_i up to at least 60 kPa, which is still low but close to the lower limit of the range of σ_i's measured near the time of breakup ($\simeq$ 100 kPa (Frankenstein 1961)).

(b) Equation [15] ignores stresses caused by forces that may be transferred at existing cracks. It is difficult to assess this effect because it depends on the (unknown) configuration of lateral restraints imposed by the channel boundaries on upstream ice sheets. It is estimated that, in the absence of restraints, this effect could show a two- to three-fold increase of the calculated σ_i.

(c) Creep effects that reduce the apparent ice strength have been ignored. The writer is not aware of creep data pertaining to the loading configuration at hand. For vertical loadings of the ice cover, creep reduces the apparent ice strength by 50% or more within a few hours of loading time (Assur 1961; Panfilov 1972).

It thus appears that Shulyakovskii's mechanism of transverse crack formation may apply but more data are needed for a definite conclusion on its validity. Another mechanism that can produce transverse cracks is the passage of water waves under the cover. Such waves can be caused by sudden releases of ice jams or rapid discharge increases. A first attempt to analyze this problem was made by Billfalk (1982), who assumed a linear water surface profile and ignored dynamic effects based on an order-of-magnitude comparison with static ones. This theory predicts crack spacings that are far too small relative to observations (Fig. 17) but the wave breaking theory needs further development before deciding on its applicability. For the present, it is noted that wave breaking would produce l_i's that are largely independent of W_i and τ but dependent on such additional parameters as initial water surface configuration, wave celerity, E_i, and γ.

Effects of ice deterioration and driving forces

Inspection of Figs. 15 and 16 suggests the following empirical equation:

$$[16] \quad W_B/W_i = 1 + C(100h_i/W_i)$$

According to [8], C should depend on σ_i/τ and dimensionless river constants. The former parameter was introduced via Shulyakovskii's (1972) mechanism of transverse crack formation. However, it was shown in the previous section that this mechanism cannot as yet be confidently accepted. Hence, σ_i/τ may be more appropriately replaced by several other dimensionless factors reflecting ice and flow properties. In general, C would be expected to decrease with increasing degree of thermal ice deterioration and possibly with increasing τ.[3]

The mechanical properties of ice at the time of breakup can vary over wide ranges and the process of thermal deterioration is not well understood at present (Frankenstein 1961; Korzhavin 1971; Butyagin 1972). Bulatov (1972) outlined a method for computing ice strength based on theoretical and experimental correlations with radiation effects; however, this paper is too general to permit application of the proposed method by others.

[3]The wave breaking mechanism, for example, should be largely independent of τ. At the same time, it is difficult to conceive instances where C would increase with increasing τ.

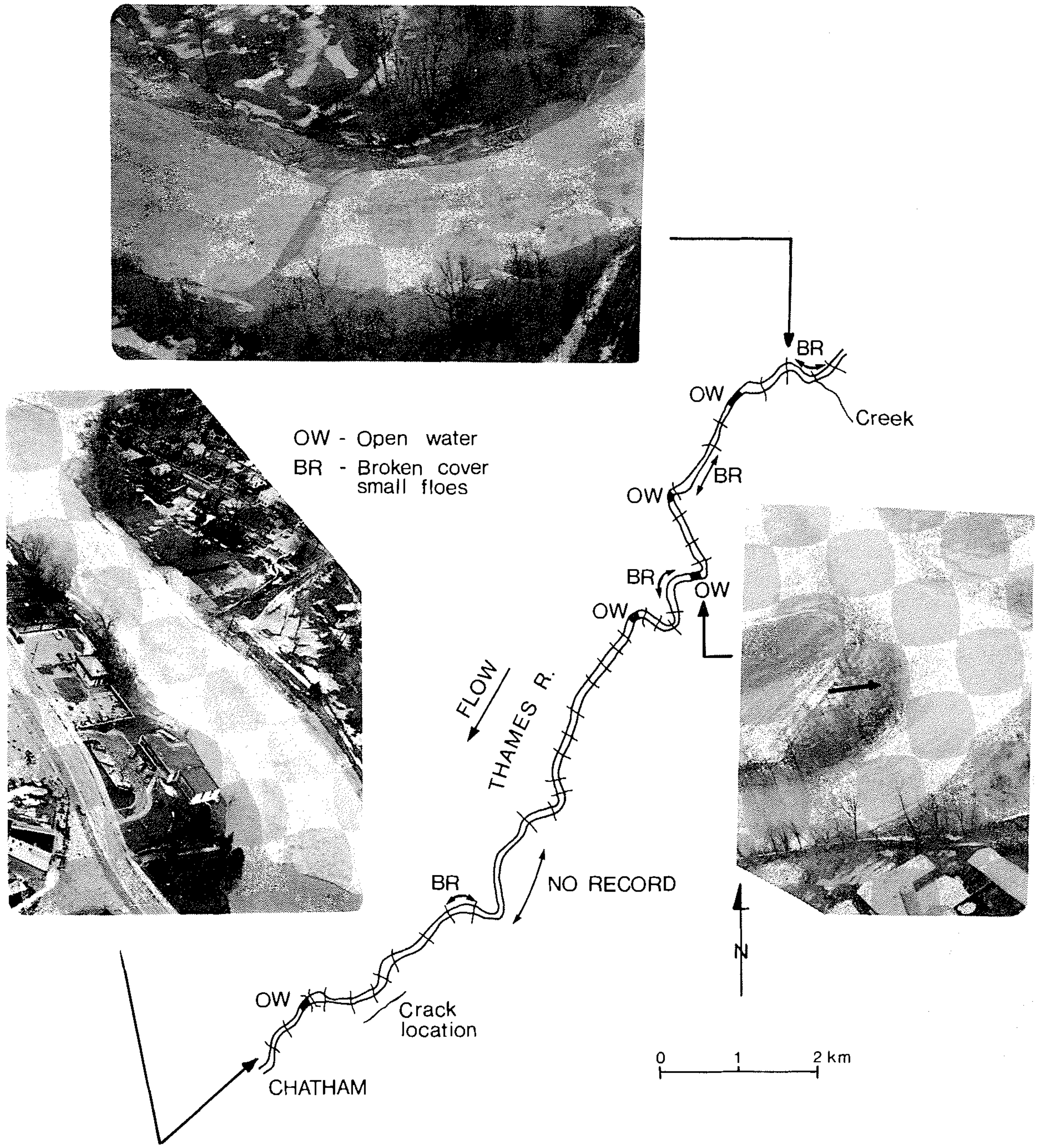

FIG. 17. Observed transverse crack pattern, Thames R. above Chatham, Mar. 17, 1982.

Ashton's (1983) analysis is similar to Bulatov's and shows that the main agent of deterioration is penetrating solar radiation, once the ice has been warmed to 0°C. Additional radiation absorption causes melting at the grain boundaries with a resulting decrease in strength. However, Ashton's theory cannot be applied to the present data because information on snow cover, albedo, and ice structure is lacking. Thus, thermal effects can only be studied at present by introducing empirical indices intended to describe weather conditions. For the Nashwaak River data, preliminary analysis indicated that both accumulated degree-days of thaw and hours of sunshine influenced the onset of breakup. To reduce the number of thermal indices as well as introduce the in-

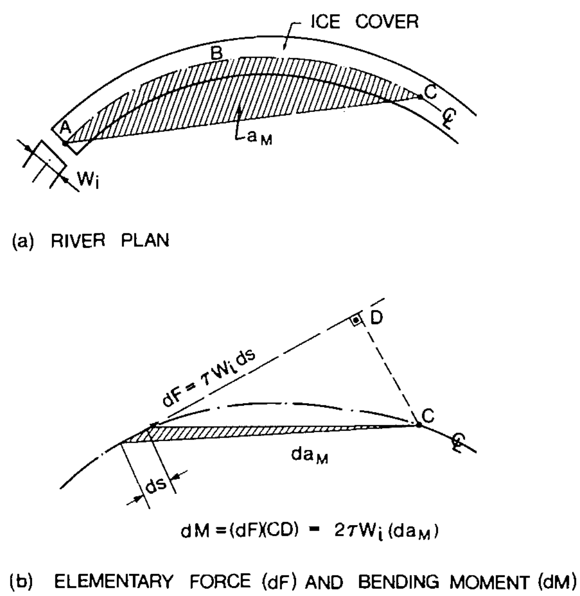

FIG. 18. Derivation of [14].

coming solar radiation, a single index, $\Sigma\ q$, was also tried and was as effective as that of the combination of degree-days and sunshine. The index $\Sigma\ q$ is a calculated heat input per unit ice cover area, accumulated to the time of breakup initiation (see Shulyakovskii (1963) and Beltaos and Lane (1982) for computational details). For the Nashwaak River data, it was subsequently found that C decreases with increasing $\Sigma\ q$, in accordance with expectation. Pertinent meteorological data were obtained from published records (Atmospheric Environment, Environment Canada—*Monthly Records*) for a nearby station and are thus considered representative of local weather conditions. "Premature" events had a value of 0.45 for C_0 (= C for $\Sigma\ q \rightarrow 0$). Analysis of the Thames River data indicated a similar but not as well defined variation of C with $\Sigma\ q$. However, there are no nearby sunshine recording stations, so that $\Sigma\ q$ values are uncertain in this case. The value of C_0 was 0.85. For the Smoky and Peace Rivers data, C_0 cannot be determined because no premature events have been observed; all that can be stated about C_0 is that it should not be less than 0.45 and 0.52, respectively. Inspection of Table 1 suggests a trend for C_0 to decrease with increasing river slope. This may be a hint for a similar dependency on τ but the latter also depends on other factors that are generally unknown for the present data.

In conclusion, it may be stated that the present results show the expected stratification with thermal ice deterioration via empirical meteorological indices. While use of any such index can be criticized on the grounds of not adequately representing the physical processes involved, the C_0 values quoted earlier are not dependent on the type of index used. For the four river sites considered herein, C_0 is between 0.45 and 0.85. This is remarkably consistent, considering the associated large variation in magnitude, hydraulics, and latitude of the respective streams. The effects of driving forces and channel plan geometry remain unclear. Progress in this regard requires additional case studies and conclusive identification of the mechanism of transverse crack formation.

Limitations

The foregoing discussion suggests several conditions under which the present model may not apply, i.e.,

(i) "Overmature" breakup events during which the ice cover largely disintegrates by thermal effects rather than breaking by mechanical action.

(ii) Reaches where the water level is strongly influenced by nearby controls. An example is the Thames River near the mouth where the stage is controlled by that of Lake St. Clair. In this reach the stage hardly rises prior to breakup initiation, so that the transverse crack and loose ice sheet pattern may not occur. Breakup is usually initiated by thermal deterioration and mechanical destruction effected by advancing ice jams.

(iii) Channel types that are significantly different from the single, meandering channels considered herein, e.g., braided, multiple islands, straight channels, etc.

Discussion

A conceptual model of river ice breakup has been developed and used as a framework for analyzing pertinent data from four gauge sites. While the data from these sites are consistent with the present model, direct observational verification (e.g., Fig. 17) has, so far, been obtained in only one of the rivers studied (Thames River). Moreover, the analysis identified several additional gaps in existing information. Improved knowledge is thus needed for the following aspects of breakup: ice cover thickness, discharge hydrograph, mechanism of thermal ice deterioration, ice cracking patterns prior to breakup initiation, and accumulation of additional case studies over representative ranges of river morphology and climate.

The present analysis focused on forecasting the onset and flooding potential of the breakup. Other things being equal, the breakup initiation stage increases with increasing freeze-up stage, ice thickness, and strength, and with decreasing channel width and slope. The flooding potential of breakup is largely governed by discharge, which dictates the potential stage of any ice jams that might occur. The present model suggests that there should be an "ice-clearing" discharge such that larger discharges will be associated with unstable, if any, jams. This places an additional limitation on the flooding potential of breakup depending on freeze-up stage, ice thickness, and channel width and slope.

A major factor facilitating the onset and progress of breakup has been identified as the available water surface width in relation to the size of separate ice sheets that form by transverse cracking. In turn, this can suggest possible means of breakup flood control in addition to commonly used methods, e.g., keeping freeze-up levels low or placing dykes some distance off the channel banks. Moreover, the present results can provide preliminary guidance in assessments of changes in breakup dates and stages caused by alterations of the hydrologic regime (e.g., hydroelectric development).

Summary and conclusions

The breakup model developed herein provides a framework for interpreting and generalizing data pertaining to breakup forecasting, which to date has been site specific. The main factor facilitating the onset and progress of breakup has been identified as the available water surface width relative to the size of separate ice sheets formed by transverse cracking. Thus, it has been possible to quantify the effects of such factors as ice cover dimensions and (partly) channel geometry. Owing to lack of data, other parameters (e.g., mechanical properties of ice and driving forces) have only been considered indirectly to elucidate trends. The mechanism of transverse cracking was examined in the light of recent observations. Bending on planes parallel to the water surface, caused by stream curvature, could account for the observed crack spacing but more evidence is needed for positive conclusions. The present model does not apply in cases of "overmature" breakup events, proximity of stage controls, and river planforms significantly different from the single meandering channel type.

Acknowledgments

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List of symbols

a_M	area between channel centre line and chord
C	dimensionless empirical coefficient
C_0	value of C for premature breakup events
E_i	Young's modulus of ice
f_c	composite friction factor of flow under an ice sheet
$f_1, f_2, \ldots$	functions
g	acceleration of gravity
h_i	ice cover thickness
h_s	snow cover thickness
H	water stage; gauge height
H'	rise in water stage above an initial value
H_B	water stage when breakup is initiated
H_F	maximum stable freeze-up stage (daily average)
H_m	maximum stage during breakup
H_0	initial water stage
I	moment of inertia per unit ice cover width
l_i	length representing longitudinal dimensions of ice sheets that form prior to breakup initiation
l_s	distance of a side crack to nearest channel bank
$L_1, L_2, \ldots$	lengths defining river plan geometry
M	bending moment
q_c	ice clearing discharge intensity
q_m	maximum breakup discharge intensity ($\simeq Q_m/W_B$)
Q_m	maximum discharge during breakup
R	average radius of curvature of an ice sheet
s_i	specific gravity of ice
S	channel slope
V	average flow velocity
W	water surface width
W_B	value of W at the stage H_B
W_F	value of W at the stage H_F
W_i	ice cover width
Y	average water depth
Y_B	value of Y at the stage H_B
Y_c	value of Y at the stage needed to dislodge the last ice sheet in a reach
Y_F	value of Y at the stage H_F
γ	unit weight of water
ΔH	uplift pressure head required to cause side cracking
θ	central angle of a curved ice sheet
$\theta_1, \theta_2, \ldots$	angles defining river plan geometry
λ	parameter with dimension of length^{-1}, characteristic of beams on elastic foundation systems
Σq, Σq_i	calculated heat inputs per unit upper and lower area (respectively) of ice cover, accumulated to the time of breakup initiation
σ_i	ice strength
τ	tangential driving force per unit ice cover area
Φ	set of parameters describing stream morphology